

Optimal Energy Shaping and Force Amplification Framework for Task-Agnostic, Biomimetic Ankle Exoskeletons

Katharine Walters¹, Gray C. Thomas² and Robert D. Gregg¹

Abstract—Task-agnostic controllers for partial-assist lower-limb exoskeletons aim to reliably mimic biological torque while seamlessly adapting to changing movement patterns. However, current approaches relying on hidden state estimators or neural networks lack explainability and safety guarantees, while force amplification methods risk instability with an inherent trade-off between sensitivity and robustness to control inputs. Energy shaping control uses a kinematic model-based framework to provide predictable, stable assistance, though its traditional passive form limits biomimetic performance. Previous work relaxed the strict passivity requirements to improve biomimicry but reduced the stability guarantees. This paper presents an optimization-based extension of the energy-shaping control framework that combines the stability benefits of energy shaping with the intuitive biomimicry of force amplification. Our framework enables controlled trade-offs between sensitivity to changing human impedance and high performance through adjustable cost contributions of force amplification and model-based terms. We provide theoretical guarantees of closed-loop stability to an invariant set under human joint impedance control, supported by empirical validation of stability characteristics of an ankle exoskeleton under varying controller passivity constraints. A study of ten able-bodied participants using bilateral ankle exoskeletons demonstrates that the biomimetic controller reduced biological ankle torque by 19.1% across various activities of daily life.

I. INTRODUCTION

Partial-assist lower-limb exoskeletons deliver a fraction of the biological joint torque normally produced by the user’s musculature, holding significant promise for a wide range of users. In able-bodied users, biomimetic torque assistance can reduce joint contact forces [1], muscle exertion [2]–[4], energy expenditure, and joint work [5]. Moreover, impaired individuals have benefited from reduced effort, improved mobility metrics (e.g., walking speed) [6], or acute joint pain reductions [7]–[9]. However, it is difficult to design exoskeleton controllers that robustly and predictably mimic biological torque while adapting to changing movement patterns in an understandable way. Without all these features, exoskeletons could apply unhelpful or unexpected torques that jeopardize user safety or trust in the device [10].

Current control approaches largely rely on estimators that leverage hidden states to infer gait parameters from kinematic

inputs. For periodic movements like steady-state walking, pre-defined torque profiles can be applied by estimating the hidden gait phase either as a function of time between heel-strikes [11]–[14] or via a nonlinear oscillator that converges to tracking the user’s periodic gait oscillation [15]. Machine learning classifiers extend these approaches by estimating hidden task states to adapt to changing movement patterns [16], [17]. Alternatively, gait phase and task-specific variables (like stride length and incline) can be continuously estimated through machine learning [18], [19] or extended Kalman filters [20] to enable continuous adaptation to transient conditions and varying terrains. However, estimators with hidden states struggle with rapid transitions and become increasingly complex with more tasks, rendering control outputs unpredictable for users and challenging to debug.

Recent work has moved toward estimating joint torque directly by learning hidden mappings between kinematics and torque with neural networks [5], [21]. This approach eliminates the need for hidden task or gait state estimation while adapting to continuously varying terrains and even tasks outside of the training data. These task-agnostic controllers for knee and hip exoskeletons have successfully reduced metabolic cost for able-bodied users. However, the output of multi-layer neural networks is difficult to explain and debug (and possibly unbounded outside the training distribution)—these “black box” learning methods obscure the underlying reasoning for their control decisions. Moreover, system performance cannot feasibly be verified across the entire spectrum of possible control inputs, and these approaches lack formal guarantees for safe interaction with the user in unpredictable real-world scenarios. These limitations motivate alternative control approaches that prioritize explainability and rely on direct connection to user intent.

Direct volitional control methods apply assistance as a function of measured user effort, creating an inherently task-agnostic control law without hidden states or “black box” learning. The user’s joint effort can be quantified through muscle activity measured with surface electromyography (EMG), where assistance is applied as a linear function of flexor and/or extensor activation [22], or via a neuromuscular model accounting for nonlinear joint torque generation dynamics [23]–[25]. However, EMG-based torque estimation is susceptible to signal variation caused by surface electrodes [26], muscle co-contraction [23], [26], and changes in effort due to exoskeleton assistance [27]. These practical limitations associated with measuring muscle activity have motivated research in mechanical sensing alternatives to estimate user effort.

Force amplification is an alternative direct volitional control strategy, where the user’s net joint torque is estimated as a function of interaction forces measured at environmental

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Katharine Walters and Robert D. Gregg are with the Department of Robotics, University of Michigan, Ann Arbor, MI 48109, USA. {kwalte, rdgregg}@umich.edu

Gray C. Thomas is with the J. Mike Walker ’66 Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843, USA. {gthomas}@tamu.edu

interfaces (e.g., the ground) [28]–[31]. This approach offers an intuitive and biomimetic control law for the ankle in particular, due to the simple kinematic chain of the ankle relative to the ground [28]. However, direct force amplification risks unstable oscillations with an inherent trade-off between sensitivity to the user’s input and robustness to variation in system model parameters [31]. The stability of such systems depends on the varying impedances of the exoskeleton, human, and environment [32]–[35]. Without trustworthy models of these systems, it is challenging to design force amplification controllers that ensure safe interaction with the user.

Energy shaping control leverages a kinematic model-based structure to provide task-agnostic assistance while guaranteeing stability without hidden states or knowledge of human or environmental impedance [36]. This control strategy posits a Lagrangian or Hamiltonian model for the closed-loop system and then derives the control law necessary to achieve it—guaranteeing energetically passive closed-loop exoskeleton behavior [37]. For example, potential energy shaping uses feedback from actuated joint angle(s) to provide gravity compensation torques, effectively altering the user’s perceived gravitational forces [36]. This passive approach can also include virtual springs and dampers [38], allowing net energy dissipation but not injection over a closed path. This passivity guarantee ensures the user remains in control of net energy growth in the coupled human-exoskeleton system, which can be easily stabilized by human joint impedance. However, strict passivity also limits torque biomimicry across tasks [38], as net energy can only be injected when switching between distinct stance and swing controllers. Biomimicry can be improved by relaxing passivity to allow the control law to depend on the unactuated global angle [3] as well as ground contact forces for tapering torque during stance-swing transitions [39]. While this relaxed framework has shown promising results for hip and knee exoskeletons [40], the compatible sensing available to a self-contained ankle exoskeleton is insufficient to accurately adapt torque between diverse activities [41]. This passivity relaxation also reduces the human input space for which the controller is provably stable [3].

This paper introduces an optimization-based task-agnostic control framework that integrates the complementary strengths of passivity-based energy shaping and direct force amplification, while addressing the limited biomimicry of the former and the stability vulnerabilities of the latter. Using representative able-bodied data (rather than device-specific data as in [5], [21]), the optimization enables a controlled trade-off between biomimetic performance and robustness to variable human impedance by changing the relative cost contributions of force amplification and model-based terms, respectively (Fig. 1). First, we extend the optimization-based energy shaping framework in [3], [41] to formally include force amplification control contributions and derive an explicit control law for a stance/swing controller for a self-contained ankle exoskeleton. Next, we theoretically analyze the passivity properties of the closed-loop system and prove stability to an invariant set in the presence of human joint impedance control, without assuming the human compensates for exoskeleton underactuation as in [3], [40]. The invariant set restricts the human-exoskeleton

system to a bounded region of system states, with its size determined by the magnitude of non-passive controller inputs (e.g., force amplification). Larger non-passive contributions expand the set and increase the potential for uncertainty or oscillatory behavior within it. While previous work [3], [40] did not constrain these non-passive inputs, our optimization framework incorporates tunable design constraints for restricting energy injection across tasks and subjects.

To validate this novel framework with constrained force amplification torque contributions, we empirically demonstrate its stability benefits over unconstrained force amplification with a bilateral, partial-assist ankle exoskeleton system. We further evaluate biomimetic controller performance and reductions in user ankle torque across ambulatory and non-ambulatory activities with $n=10$ able-bodied participants. By reducing biological ankle torques by 19.1%, task-agnostic ankle exoskeletons have the potential to reduce contractile forces on the articular surfaces of the joint [1], [42], a key factor in osteoarthritic pain and overuse injuries [43]–[45].

II. ENERGY SHAPING FRAMEWORK

This section presents the system model and extends the optimal energy shaping framework introduced in [3] to combine force amplification and model-based approaches. We focus the model on the ankle to derive an energy shaping control law for an ankle exoskeleton and theoretically investigate the passivity and stability of the closed-loop system.

A. System Model

The human-exoskeleton system is modeled with two links representing the shank and foot joined by a revolute joint representing the ankle (Fig. 1, left). The Cartesian position of the ankle given by (p_x, p_y) is defined with respect to an inertial reference frame at the ground coincident with the heel at heel strike. By defining the base of the floating kinematic chain at the ankle, we can easily model the system with respect to the global shank angle, ψ , defined relative to the vertical. The origin of the shank is defined at the ankle, and the relative ankle angle is given by θ . The four degree-of-freedom (DOF) system has generalized coordinates $q = [p_x, p_y, \psi, \theta]^T \in \mathbb{R}^4$. The body interaction forces resulting from unmodeled human dynamics proximal to the shank influence the system dynamics through the wrench $F = [f_x, f_y, \tau_z]^T \in \mathbb{R}^3$ applied at the proximal shank-tip. The conjugate momenta is given by $p = M(q)\dot{q} \in \mathbb{R}^4$ for the positive-definite inertia matrix $M(q) \in \mathbb{R}^{4 \times 4}$ and velocity vector $\dot{q} \in \mathbb{R}^4$.

Port-Hamiltonian systems offer a unified approach for modeling the dynamics of this complex system by introducing non-conservative external “ports” through which energy is exchanged with the environment or an external agent (e.g., the human or the exoskeleton) [46]. Each port consists of an effort input (i.e., a force or torque) and corresponding flow output (i.e., a velocity). Within this framework, the dynamics are derived from the Hamiltonian $H = \frac{1}{2}p^T M^{-1}(q)p + V(q)$, where the quadratic term is kinetic energy and $V(q)$ is potential energy, giving the equations of motion

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{4 \times 4} & I_{4 \times 4} \\ -I_{4 \times 4} & 0_{4 \times 4} \end{bmatrix} \nabla H + \begin{bmatrix} 0 \\ \tau_{\text{hum}} + A(q)^T \lambda_F + Bu \end{bmatrix}, \quad (1)$$

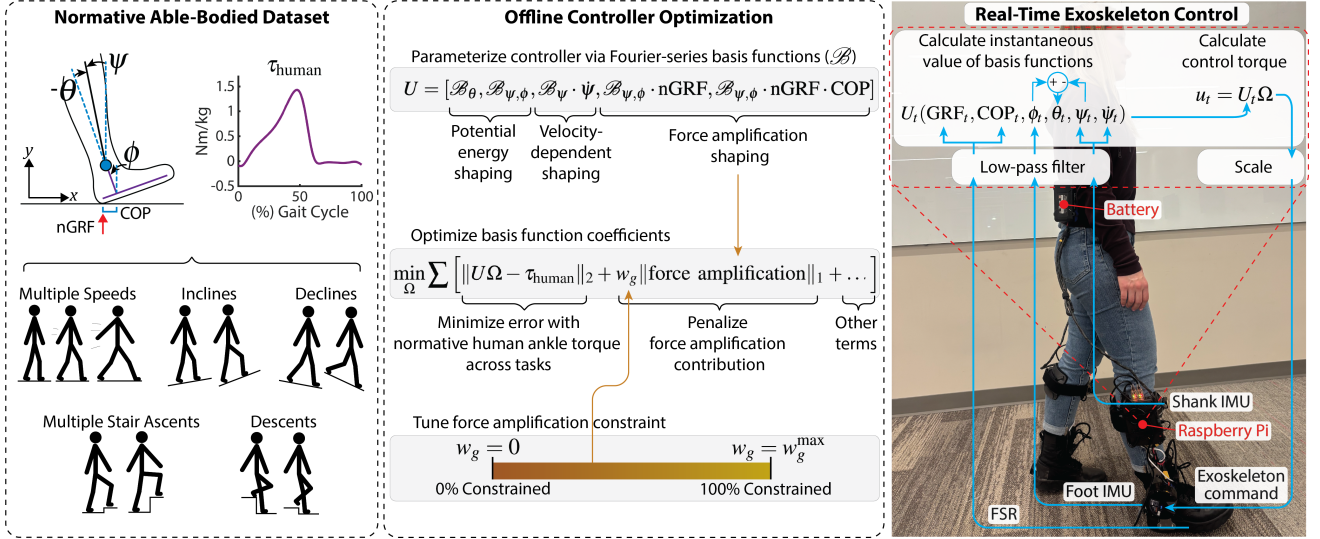


Fig. 1. Block diagram depicting the controller design and implementation. (Left) The controller is trained on time-normalized strides from multiple able-bodied subjects across ADLs from a dataset. (Middle) The controller is parameterized by Fourier-series basis functions of system angles. The basis function coefficients are optimized to minimize the error between the control torque and normative torque while penalizing contributions from force amplification shaping, with other terms for smooth fitting and training detailed in Section III. The force amplification contribution is penalized with scalar weight $w_g \in [0, w_g^{\max}]$, where $w_g^{\max}$ is a finite positive scalar. (Right) In real-time control, the instantaneous value of the basis functions is calculated via IMU and FSR signals. The mass-normalized control torque is scaled by a user-specific assistance fraction and the user's mass before being commanded to the actuator.

with the lower-right term aggregating the external inputs. The human torque input $\tau_{\text{hum}} = Bv + J(q)^T F$ aggregates the human ankle torque $v \in \mathbb{R}$, mapped into the 4-dimensional coordinate space by the selector matrix $B = [0_{1 \times 3}, 1]^T$, and the body interaction forces, mapped into the equation through the proximal shank-tip Jacobian $J(q) \in \mathbb{R}^{3 \times 4}$. The exoskeleton torque $u \in \mathbb{R}$ is mapped into the equation through the same selector matrix B . The ground reaction forces (GRFs), $\lambda_F \in \mathbb{R}^c$, are mapped into the equation through constraint matrix $A(q) \in \mathbb{R}^{c \times 4}$, where c is the number of constraints. The constraint matrix $A(q) = \partial_q a(q)$ satisfies $A(q)\dot{q} = 0$, where $a(q) = 0$ defines holonomic contact constraints on the floating kinematic chain with respect to the IRF. These contact constraints (modified from [3]) are defined relative to the base of the floating kinematic chain at the ankle and enforce a no-slip condition for heel contact, flat foot, and toe-off configurations. The same system model describes stance and swing phases, where swing phase has no contact constraints.

This port-Hamiltonian system offers a physically interpretable formulation for combining model-based and force-amplification control approaches by explicitly accounting for energy exchange with each control contribution through well-defined ports. The human influences the system dynamics through a human torque input port with effort τ_{hum} and flow $\dot{q}$. Force amplification shapes energy at the environmental interaction (i.e., GRF) port with effort λ_F and flow $A(q)\dot{q}$, where the open-loop constraint forces do no work (i.e., $\dot{q}^T A(q)^T \lambda_F = 0$). Model-based energy shaping applies configuration-dependent torque through a port with effort u and flow $B^T \dot{q}$. This approach unifies model-based and force-amplification control strategies in a consistent formulation.

B. Matching Conditions and Energy Shaping Control Law

To derive the exoskeleton control law, we assume a closed feedback loop for the exoskeleton input u , while the hu-

man inputs v and F remain open-loop in the Hamiltonian system. We consider a desired closed-loop Hamiltonian $\tilde{H}(p, q) = \frac{1}{2} p^T \tilde{M}^{-1}(q) p + \tilde{V}(q)$, where $\tilde{M}$ is the desired inertia matrix and $\tilde{V} = V + \hat{V}$ is the desired potential energy with shaping term $\hat{V}$. We set $\tilde{M} = M$ to avoid requiring noisy acceleration feedback or unrealistic knowledge of human ankle torque and body interaction forces in the control law. The desired closed-loop dynamics are

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{4 \times 4} & I_{4 \times 4} \\ -I_{4 \times 4} & J_2 \end{bmatrix} \nabla \tilde{H} + \begin{bmatrix} 0 \\ \tau_{\text{hum}} + \tilde{A}(q)^T \tilde{\lambda}_F + B_x u_x \end{bmatrix}, \quad (2)$$

where $\tilde{\lambda}_F$ is the closed-loop GRF vector mapped into the equation by the closed-loop constraint matrix $\tilde{A} = A + \hat{A}$, with constraint shaping term $\hat{A}$. Following the method introduced in [3], we add an external input $u_x \in \mathbb{R}$ (to be explained later) that is restricted to act only on the unactuated global coordinate ψ by the selector matrix $B_x = [0, 0, 1, 0]^T$. The skew-symmetric matrix $J_2 \in \mathbb{R}^{4 \times 4}$ provides extra DOFs for velocity-dependent shaping through the interconnection structure (corresponding to gyroscopic forces that do no mechanical work) in the interconnection and damping assignment passivity-based control method (IDA-PBC) [2].

To synthesize a control torque u that achieves the desired closed-loop system behavior, we begin by equating the open-loop (1) and closed-loop (2) dynamics. This yields the following expression for the control input:

$$Bu = -(\partial_q \tilde{H})^T + (\partial_q H)^T + J_2(\partial_p H)^T + \tilde{A}^T \tilde{\lambda}_F - A^T \lambda_F + B_x u_x. \quad (3)$$

Here, each term represents either a difference in the desired and actual system energies or the effect of external forces and control inputs. However, not all DOFs are directly actuated by the exoskeleton—some are unactuated and cannot be controlled by u . To ensure that the closed-loop system is physically achievable, we must guarantee that the unactuated

dynamics are compatible with the desired energy shaping. This requirement is formalized by the so-called *matching condition*.

To derive the matching condition, we project both sides of (3) onto the subspace orthogonal to the actuated directions:

$$0 = B^\perp [-(\partial_q \tilde{H})^T + (\partial_q H)^T + J_2 M^{-1} p + \tilde{A}^T \tilde{\lambda}_F - A^T \lambda_F + B_x u_x], \quad (4)$$

where $B^\perp$ is a full-rank left annihilator of B satisfying $B^\perp B = 0$. Intuitively, this equation states that, for the closed-loop dynamics to be realizable, the open-loop and closed-loop dynamics must “match” wherever actuation is unavailable. This matching condition is satisfied by setting the first three (unactuated) rows of the vector equation (3) equal to zero.

To find the exoskeleton input u that achieves this matching condition, we first consider the force amplification terms that shape the system energy during foot contact with the ground. Recall that the environmental interaction influences the open-loop system through a port with effort (i.e., input) λ_F and flow $A(q)\dot{q}$. The same environmental interaction port influences the closed-loop system with effort $\tilde{\lambda}_F$ and flow $\tilde{A}(q)\dot{q}$. As the environment does not change, we set $\lambda_F = \tilde{\lambda}_F$. Thus, the GRF shaping terms simplify to $\tilde{A}^T \lambda_F - A^T \lambda_F = \hat{A}^T \lambda_F$, where $\hat{A}^T = \tilde{A}^T - A^T$. We shape the behavior of the environmental interaction port on the closed-loop system dynamics by choosing the constraint shaping term $\hat{A}$ as

$$\begin{aligned} \hat{A}_{\text{heel}}(q) &= \hat{A}_{\text{toe}}(q) = \begin{bmatrix} 0 & 0 & 0 & -\varepsilon \cdot \sin(\gamma) \\ 0 & 0 & 0 & \varepsilon \cdot \cos(\gamma) \end{bmatrix}, \\ \hat{A}_{\text{flat}}(q) &= \begin{bmatrix} 0 & 0 & 0 & -\varepsilon \cdot \sin(\gamma) \\ 0 & 0 & 0 & \varepsilon \cdot \cos(\gamma) \\ 0 & 0 & 0 & \eta \end{bmatrix}, \end{aligned} \quad (5)$$

where γ is the ground inclination. The first three columns corresponding to the three unactuated coordinates are zero, satisfying the matching condition. The scale factors η and ε are chosen to shape the energy about the actuated joint with respect to the moment GRF component (τ_θ) and the GRF component normal to the ground (nGRF), respectively, where $\text{nGRF} = -\text{GRF}_x \sin(\gamma) + \text{GRF}_y \cos(\gamma)$. We focus on shaping τ_θ and nGRF as we can approximately measure these quantities via a pressure-sensing insole (see Section IV-B).

Next, we consider the terms related to the shaped potential energy, where $-(\partial_q \tilde{H})^T + (\partial_q H)^T = -\partial_q \hat{V}$. We choose $\hat{V}$ such that $\frac{\partial \hat{V}}{\partial p_x} = 0$ and $\frac{\partial \hat{V}}{\partial p_y} = 0$, but requiring $\frac{\partial \hat{V}}{\partial \psi} = 0$ (i.e., choosing a shaped potential energy depending only on the actuated ankle angle θ) would restrict the controller to virtual spring-like behavior. To increase the flexibility of the controller to produce biomimetic torques, we allow $\hat{V}$ to depend on both the unactuated global coordinate ψ and the actuated coordinate θ , as in [3], [41]. The nonzero $\frac{\partial \hat{V}}{\partial \psi}$ does not immediately satisfy the matching conditions, which will be addressed later.

The $J_2 M^{-1} p$ term in (4) shapes the system energy with respect to velocity. As J_2 must be skew-symmetric, we choose

$$J_2 = \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 2} \\ 0_{2 \times 2} & \begin{bmatrix} 0 & \kappa \\ -\kappa & 0 \end{bmatrix} \end{bmatrix}, \quad (6)$$

where $\kappa \in \mathbb{R}$ scales the velocity-dependent shaping. The $J_2 M^{-1} p$ term does not shape the first two coordinates, sat-

isfying the matching condition for p_x and p_y . However, this velocity-dependent shaping does assume torque is applied to the unactuated global DOF ψ as a function of θ , which also does not immediately satisfy the matching condition.

The potential energy and velocity-dependent shaping about the unactuated global coordinate cannot be matched directly. We therefore define a fictitious external input, u_x , which cancels the torque acting on this coordinate resulting from $\frac{\partial \hat{V}}{\partial \psi} \neq 0$ and velocity-dependent shaping, i.e.,

$$u_x = \frac{\partial \tilde{H}}{\partial \psi} - \frac{\partial H}{\partial \psi} - \kappa \dot{\theta}. \quad (7)$$

With this external input, the target system remains passive, though this target system cannot be perfectly realized as the external input cannot be physically applied. Thus, we can only approximately achieve the desired closed-loop Hamiltonian dynamics. This passivity relaxation permits power ($u_x \dot{\psi}$) to “leak” into and out of the physical system, allowing the exoskeleton input u to do positive or negative work as a function of the unactuated potential energy and shank velocity. The explicit definition of this fictitious input enables us to formally characterize the impact of this energy injection on the stability of the closed-loop system and constrain its magnitude in the subsequent control optimization.

Thus, a feasible control law satisfying the relaxed matching conditions is

$$u = B^+ [-(\partial_q \tilde{H})^T + (\partial_q H)^T + J_2 M^{-1} p + \tilde{A}^T \lambda_F], \quad (8)$$

where $B^+ = (B^T B)^{-1} B^T$ is the left pseudo-inverse of B .

C. Passivity Analysis

We theoretically investigate the input-output passivity of the closed-loop system as follows:

Proposition 1: The closed-loop system in (2) is passive with three input ports: 1) the human torque with effort τ_{hum} , 2) the external input with effort $B_x u_x$, and 3) the GRFs with effort $\tilde{A}^T \lambda_F$ (where $\tilde{\lambda}_F = \lambda_F$), where each port has flow $\dot{q}$.

Proof 1: Consider the storage function $E = \tilde{H} = H + \hat{V}$, where $(\partial_p \tilde{H})^T = (\partial_p H)^T = \dot{q}$, $\partial_q \tilde{H} = \partial_q H + \partial_q \hat{V}$, and $\tilde{A}^T = A^T + \hat{A}^T$. The time derivative of E is

$$\begin{aligned} \dot{E} &= (\partial_q H) \dot{q} + (\partial_q \hat{V}) \dot{q} + (\partial_p H) \dot{p} \\ &= (\partial_q H) \dot{q} + (\partial_q \hat{V}) \dot{q} + (\partial_p H) \left[-(\partial_q \tilde{H})^T - (\partial_q \hat{V})^T + \right. \\ &\quad \left. J_2 (\partial_p H)^T + \tau_{\text{hum}} + \tilde{A}^T \lambda + B_x u_x \right] \\ &= (\partial_p H) J_2 (\partial_p H)^T + (\partial_p H) \tau_{\text{hum}} + (\partial_p H) \tilde{A}^T \lambda_F + (\partial_p H) B_x u_x \\ &= \dot{q}^T \tau_{\text{hum}} + \dot{q}^T A^T \lambda_F + \dot{q}^T \hat{A}^T \lambda_F + \dot{q}^T B_x u_x \\ &= \dot{q}^T \tau_{\text{hum}} + \dot{q}^T \tilde{A}^T \lambda_F + \dot{q}^T B_x u_x, \end{aligned} \quad (9)$$

where the quadratic form of skew-symmetric matrix J_2 is zero, and the motion of the foot is constrained relative to the IRF, i.e., $A\dot{q} = 0$, such that the constraint $\dot{q}^T A^T \lambda_F$ does no work. As the shaped constraint matrix $\hat{A}$ is modified with respect to the original constraint matrix A , the net power through the shaped GRF port $\dot{q}^T \hat{A}^T \lambda_F$ is nonzero. Thus, the closed-loop system is passive with respect to the three input ports. $\square$

Hence, the human controls energy growth through the port $\dot{q}^T \tau_{\text{hum}}$, which varies continuously throughout the gait cycle. The shaped GRF port $\dot{q}^T \hat{A}^T \lambda_F$ enables continuous energy injection from the exoskeleton while the foot is in contact with the ground (i.e., force amplification). The power from the fictitious external input, $\dot{q}^T B_x u_x$, allows energy to “leak” into and out of the target system, corresponding to the exoskeleton absorbing/injecting energy in the physical system. We consider stance and swing phases separately, where energy injection via the power leak port is continuous within each discrete phase.

The binary switch between stance and swing also allows instantaneous energy injection at the transition, e.g., changing the equilibrium of a virtual spring. As the user volitionally controls their kinematics, we assume that the user controls the timing of transitions and thus energy injection. Moreover, the user’s control over the system configuration at the transition grants them some indirect control over the magnitude of energy injection. Since the controller is designed to provide assistance proportional to the user’s joint effort, the energy injected would also scale predictably and appropriately. Importantly, the energy injection (e.g., the virtual spring set-point) resets at every transition, preventing cumulative or runaway energy even during rapid successive transitions. This architecture preserves user control over discrete energy injection.

D. Stability Analysis

Our closed-loop system allows the exoskeleton to continuously inject energy via the shaped GRF and power leak ports (9). We theoretically investigate system stability under the worst-case magnitude of these external control inputs. Ref. [3] introduced the potential energy power leak and proved system stability with a human model combining an impedance term and a gravity compensation term. However, this approach does not easily extend to include our unmodeled input terms from the force amplification contribution.

As human perturbation responses are well-described by an impedance model [47]–[49], we use a simple impedance-controlled human input:

$$\tau_{\text{hum}} = -K_p' e - K_d \dot{e} + G(\bar{q}), \quad (10)$$

with constant positive-definite stiffness $K_p' \in \mathbb{R}^{4 \times 4}$, constant positive-definite damping $K_d \in \mathbb{R}^{4 \times 4}$, $e = q - \bar{q}$ representing the error between the human’s current configuration q and the human’s constant setpoint $\bar{q}$, and $G(\bar{q}) = \partial_q \tilde{V}^T|_{q=\bar{q}}$ representing the human’s compensation of shaped gravity to achieve equilibrium. To account for the effective linear spring rate due to shaped gravity, $\nabla G(q)|_{q=\bar{q}}$, which is potentially indefinite, we define the total effective spring rate K_p as

$$K_p = K_p' + \nabla G(q)|_{q=\bar{q}}. \quad (11)$$

To stabilize our equilibrium, we require that $K_p \succ 0$.

As our goal is to analyze the controller’s stability properties and not precisely model human neuromechanics, this linearized human model provides a useful local approximation of the human response within the feedback control loop for small deviations around $\bar{q}$. While the true human neuromechanical

response is nonlinear and may exhibit behaviors not captured by the linearized model, particularly under large perturbations or during transitions between distinct set-points, local stability of the human-exoskeleton system is a necessary condition for robust operation. Hence, linearized analysis provides critical insight into controller behavior that remains relevant for a real-world nonlinear human input.

The following Theorem holds locally around the human impedance model’s equilibrium vector $\bar{q}$, where the mass/inertia matrix M is approximately constant [47]–[49] and $\bar{J} = J_2(\bar{q})$ denotes the velocity-dependent shaping at $\bar{q}$.

Theorem 1: Assume the mass/inertia matrix M is constant. The system (1) under exoskeleton controller (8) and human spring-damper behavior (10) satisfying

$$K_p \succ \alpha^2 M, \quad (12)$$

$$\alpha (4M + (K_d - \bar{J})^T K_p^{-1} (K_d - \bar{J})) \prec 4K_d, \quad (13)$$

with scalar $\alpha > 0$, is stable to an invariant set—a locally attractive set of states within which the system will remain—bounded by a function of the magnitude of the control inputs λ_F and u_x , acting on the system through external ports (9).

With proof of Theorem 1 in the Appendix, we show the size of the invariant set depends on the relative magnitudes of the maximum non-passive control contribution ($\hat{A}^T \lambda_F + B_x u_x$) and the minimum human impedance torque (described by M , K_p , and K_d). This result aligns with previous work on direct force amplification control, where system stability depends on varying impedances of the exoskeleton, human, and the environment [27], [32]–[35]. As we cannot determine the change in system energy within the invariant set, there is a potential for oscillatory behavior within the set. We seek to improve overall stability by minimizing the size of this indeterminate region. To accomplish this, we reformulate our controller as an optimization problem with constraints on the maximum non-passive control contribution.

III. DATA-DRIVEN CONTROLLER OPTIMIZATION

The feasible control law (8) satisfying the relaxed matching conditions can be rewritten as

$$u = \frac{\partial H - \partial \tilde{H}}{\partial \theta} - \kappa \cdot \psi + \hat{B} \hat{A}^T \lambda_F. \quad (14)$$

However, directly solving this partial differential equation to obtain an analytical solution for the exoskeleton control law that optimally predicts biomimetic torques across various tasks (while also restricting energy injection) is impractical. In this section, we redefine the control law as a linear combination of basis functions and leverage data-driven optimization to find coefficients that produce biomimetic torques across tasks in a normative dataset. The controller optimization formulation allows the designer to constrain control torque contributions from non-passive elements, enabling the designer to optimally regulate the trade-offs between stability and biomimicry.

A. Control Law Redefined with Basis Functions

We choose trigonometric basis functions of ψ and θ (sometimes denoted with respect to $\phi = \theta + \psi$) derived from the

lowest frequency terms in the Fourier series. This choice of basis functions is appropriate to capture the periodic relationships between joint kinematics and kinetics during ambulation while generalizing to untrained, non-steady tasks (see Section VI-A). Notably, these basis functions are also bounded by construction, helping to mitigate potential risks of energy injection from non-passive control elements.

We define two functions mapping the system configuration to a vector of trigonometric functions, beginning with $\mathcal{H}_\theta(q) : \mathbb{R}^4 \mapsto \mathbb{R}^{1 \times 5}$:

$$\mathcal{H}_\theta = [\frac{1}{2}\theta^2, -\cos(\theta), \sin(\theta), -\frac{1}{2}\cos(2\theta), \frac{1}{2}\sin(2\theta)].$$

We define $\mathcal{H}_{\psi,\phi}(q) : \mathbb{R}^4 \mapsto \mathbb{R}^{1 \times 11}$ as

$$\mathcal{H}_{\psi,\phi} = [\theta, \theta\psi, \theta\sin(\psi), \theta\cos(\psi), \theta\sin(2\psi), \theta\cos(2\psi), \theta\psi + \frac{1}{2}\theta^2, -\cos(\phi), \sin(\phi), -\frac{1}{2}\cos(2\phi), \frac{1}{2}\sin(2\phi)].$$

These distinct mappings are engineered to allow manipulation of controller passivity properties, as discussed later.

We leverage these vectors of basis functions to define the change in potential energy, $\hat{V} = \mathcal{H}_\theta \omega_\theta + \mathcal{H}_{\psi,\phi} \omega_{\psi,\phi}$, where $\omega_\theta \in \mathbb{R}^{5 \times 1}$ and $\omega_{\psi,\phi} \in \mathbb{R}^{11 \times 1}$ are vectors of constant coefficients such that $\hat{V}$ is a scalar. We take the derivative with respect to the actuated coordinate to find the ankle moment due to the change in potential energy,

$$\frac{\partial \hat{V}}{\partial \theta} = \mathcal{B}_\theta \omega_\theta + \mathcal{B}_{\psi,\phi} \omega_{\psi,\phi}, \text{ with}$$

$$\begin{aligned} \mathcal{B}_\theta &= [\theta, \sin(\theta), \cos(\theta), \sin(2\theta), \cos(2\theta)] \\ \mathcal{B}_{\psi,\phi} &= [1, \psi, \sin(\psi), \cos(\psi), \sin(2\psi), \cos(2\psi), \\ &\quad \phi, \sin(\phi), \cos(\phi), \sin(2\phi), \cos(2\phi)], \end{aligned}$$

where basis functions $\mathcal{B}_{\psi,\phi}$ were previously used to provide task-agnostic plantarflexion assistance with an optimal energy shaping controller for a uni-directional ankle exoskeleton [41]. As $\mathcal{B}_\theta$ is a function of only the actuated ankle angle, the scaled basis functions produce torque as a nonlinear virtual spring about the ankle. The second term produces control torque as a function of the unactuated global coordinate. We acknowledge that fundamental kinematic information is shared in the vectors $\mathcal{B}_\theta$ and $\mathcal{B}_{\psi,\phi}$ due to the definition of $\phi = \theta + \psi$.

We define the velocity-dependent control torque contribution as a function of the system angles. Let $\kappa = \mathcal{B}_\psi \omega_\psi$, where

$$\mathcal{B}_\psi(q) = [\psi, \sin(\psi), \sin(2\psi)],$$

and $\omega_\psi \in \mathbb{R}^{3 \times 1}$ is a vector of constant coefficients. Pilot testing revealed that the high-velocity swing phase is susceptible to over-fitting to the smooth training data, which can result in errors in torque signs or magnitudes with real-time signal noise and inter-subject variability. We deliberately restricted these basis functions to depend only on ψ to achieve more consistent dorsiflexion torque magnitudes during swing across subjects.

The force amplification term $\widehat{BA}^T \lambda_F$ presents implementation challenges for a self-contained ankle exoskeleton. While true GRFs cannot be measured in real-time for such systems, a pressure-sensing insole can reasonably estimate the GRF component normal to the ground (nGRF). The moment GRF

component about the ankle joint (τ_θ) can be approximated as the product of the nGRF and the center of pressure (COP), with the COP defined as a distance along the bottom of the foot relative to the ankle. Using the GRF input scaling terms ε and η defined in (5), we approximate the GRF shaping as

$$\widehat{BA}^T \lambda_F = \varepsilon \cdot \text{nGRF} + \eta \cdot \tau_\theta \approx \varepsilon \cdot \text{nGRF} + \eta \cdot \text{nGRF} \cdot \text{COP}.$$

We shape the force amplification torque as a function of the system angles, where $\varepsilon = \mathcal{B}_{\psi,\phi} \omega_{\text{nGRF}}$ and $\eta = \mathcal{B}_{\psi,\phi} \omega_{\tau_\theta}$ with coefficient vectors $\omega_{\text{nGRF}} \in \mathbb{R}^{11 \times 1}$ and $\omega_{\tau_\theta} \in \mathbb{R}^{11 \times 1}$.

The GRF shaping control contribution is inherently limited to the stance phase of gait, and we seek to allow flexibility for kinematic-based shaping to apply separately to stance and swing phases. We introduce a scalar g that enables binary switching between stance and swing, where $g = 1$ in stance and $g = 0$ in swing. The signal g is low-pass filtered for smooth switching. This separate stance/swing shaping approach is particularly valuable for enabling biomimicry under restricted GRF contributions, especially at the ankle where torque magnitudes vary significantly between stance and swing phases.

Thus, the basis-parameterized control law is given by

$$\begin{aligned} u &= g(\mathcal{B}_\theta \omega_{\theta,\text{st}} + \mathcal{B}_{\psi,\phi} \omega_{\psi,\phi,\text{st}} + \mathcal{B}_\psi \omega_{\psi,\text{st}} \cdot \dot{\psi}) + \\ &\quad (1-g)(\mathcal{B}_\theta \omega_{\theta,\text{sw}} + \mathcal{B}_{\psi,\phi} \omega_{\psi,\phi,\text{sw}} + \mathcal{B}_\psi \omega_{\psi,\text{sw}} \cdot \dot{\psi}) + \\ &\quad + \mathcal{B}_{\psi,\phi} \omega_{\text{nGRF}} \cdot \text{nGRF} + \mathcal{B}_{\psi,\phi} \omega_{\tau_\theta} \cdot \text{nGRF} \cdot \text{COP} \\ &= U(q, \dot{q}, \text{nGRF}, \text{COP}) \Omega, \end{aligned} \tag{15}$$

where $\Omega \in \mathbb{R}^{60 \times 1}$ is the basis coefficient vector.

B. Coefficient Optimization

We formulate the control law defined in (15) as an optimization problem to find the optimal coefficient vector Ω that minimizes the error between control torque and human torque across subjects and tasks in a normative dataset. Passivity relaxation is regulated via designer-weighted constraints on control contributions from non-passive elements. The optimization problem is defined as

$$\begin{aligned} \text{minimize}_{\Omega} \quad & \sum_j \left[C_j + \Lambda \|W_\Lambda \Omega\|_1 + w_1 S_{1,j} + w_2 S_{2,j} \right. \\ & \quad \left. + \frac{w_g}{m} (G_{1,j} + G_{2,j}) + \frac{w_p}{m} (P_{1,j} + P_{2,j}) \right], \\ \text{subject to} \quad & C_j = (U_j \Omega - Y_j)^T W_{C,j} (U_j \Omega - Y_j), \\ & S_{1,j} \geq -Y_j^T U_j \Omega, \quad S_{1,j} \geq 0, \\ & S_{2,j} \geq -B_{S_j}^T (U_j \Omega - B_{S_j} \tau_{\text{sw}}), \quad S_{2,j} \geq 0, \\ & G_{1,j} \geq \|(\mathcal{B}_{\psi,\phi})_j \omega_{\text{nGRF}}\|_1, \\ & \frac{G_{2,j}}{|\text{COP}_{\text{worstcase}}|} \geq \|(\mathcal{B}_{\psi,\phi})_j \omega_{\tau_\theta}\|_1, \\ & \frac{P_{1,j}}{|\dot{\theta}_{\text{worstcase}}|} \geq \left\| g(\mathcal{B}_\psi)_j \omega_{\psi,\text{st}} \right\|_1 \\ & \quad + \left\| (1-g)(\mathcal{B}_\psi)_j \omega_{\psi,\text{sw}} \right\|_1, \\ & P_{2,j} \geq \left\| g \frac{\partial(\mathcal{H}_{\psi,\phi})_j}{\partial \psi} \omega_{\psi,\phi,\text{st}} \right\|_1 \\ & \quad + \left\| (1-g) \frac{\partial(\mathcal{H}_{\psi,\phi})_j}{\partial \psi} \omega_{\psi,\phi,\text{sw}} \right\|_1, \quad \forall j, \end{aligned}$$

where j denotes the task, $w_1, w_2, w_g, w_p \in \mathbb{R}_0^+$ are designer-tuned weights, $\tau_{sw} \in \mathbb{R}_0^+$ is a minimum swing dorsiflexion torque, $W_{C,j} \in \mathbb{R}^{m \times m}$ is a diagonal weight matrix, $B_{S,j} \in \{0, 1\}^{m \times 1}$ is a selector matrix, $\text{COP}_{\text{worstcase}} \in \mathbb{R}$ and $\dot{\theta}_{\text{worstcase}} \in \mathbb{R}$ are constants determined from normative data, and $Y_j \in \mathbb{R}^m$ is the mass-normalized normative torque for m data points for each task. The controller is optimized using time-normalized data points from the intra-subject average steady-state stride for $n=12$ able-bodied subjects [50]. The optimized tasks are 0.5 m/s, 1 m/s, and 1.5 m/s walking on level ground, 5.2° and 11° degree ramp incline and decline, and 4 inch and 7 inch stair ascent and descent. The optimization problem was solved using the MATLAB convex optimization package `cvx` [51].

The first term, C_j , minimizes the quadratic error between the control torque and reference human torque, where minimizing error for certain tasks or phases within tasks can be prioritized through scalar weights assigned in diagonal matrix W_C . As the most frequent activity of daily life (ADL) [52], the stance phase for walking tasks is weighted higher than stance phase for all other tasks. Swing phase for all tasks is assigned a higher weight than stance phase to achieve appropriate dorsiflexion assistance, as dorsiflexion torque magnitudes are small compared to plantarflexion. The second term L_1 -regularizes the optimized coefficients Ω with identity weight matrix W_Λ to encourage sparsity and minimize over-fitting, which is modulated by the scalar weight $\Lambda \in \mathbb{R}^+$.

The terms $S_{1,j}$ and $S_{2,j}$ enhance the robustness of optimized coefficients against control torques of incorrect sign, with each constraint weighted according to designer-tuned parameters w_1, w_2 . If the control torque $U_j \Omega$ and human torque Y_j have opposite signs, the variable $S_{1,j}$ takes on a positive value that the optimization seeks to minimize. If there are no incorrectly signed torques, the product of control torque and human torque takes on a negative value and the value of the objective function is not impacted as $S_{1,j}$ is constrained to be non-negative. The term $S_{2,j}$ enforces a small minimum dorsiflexion torque, τ_{sw} , during swing phase selected by logical vector $B_{S,j}$ to ensure user safety and controller robustness against signal variations. This constraint addresses two vulnerabilities during swing: the tendency for over-fitting to smooth training data at high velocities, and the potential unreliability of training data for the swing ankle (located at the end of the inverse dynamics kinematic chain).

The remaining terms in the cost function allow us to constrain control torque contributions from elements without passivity guarantees. Recall from (9) that the exoskeleton can inject or remove energy through the GRF shaping port and the power leak port that compensates for the unactuated global coordinate. Given that ports are described via an effort and a flow (i.e., velocity), restricting the effort input at the non-passive GRF and power leak ports will reduce exoskeleton energy injection (i.e., power) without subject or task-specific knowledge of velocity.

The term $(G_{1,j} + G_{2,j})$ restricts effort related to the GRF shaping port, where increasing weight w_g reduces the extent of passivity relaxation. A sufficiently large w_g will restrict the energy injection to zero across subjects and tasks, denoted $w_g^{\max}$. The value of $w_g^{\max}$ depends on the relative magnitudes

of other terms in the cost function; for instance, an increase in the quantity of training strides will increase $w_g^{\max}$. In this work, $w_g^{\max} = 24$. The term G_1 minimizes the 1-norm of the weighted basis functions scaling nGRF across all data points. The constraint assumes the worst case, where the *mass-normalized* nGRF = 1, to minimize the energy injected through the GRF port without knowledge of specific nGRF trajectories. The term G_2 minimizes the 1-norm of the weighted basis functions scaling the approximate ankle moment. We again set nGRF = 1, and the weighted basis functions are further scaled by a worst case COP (i.e., COP located at the toe) such that the constraint restricts the energy injection for any reasonable COP trajectory.

The term $(P_{1,j} + P_{2,j})$ restricts effort related to the power leak port. The optimization seeks to minimize the power leak torque u_x (7) via the scalar weight w_p . The term $P_{1,j}$ minimizes the velocity-dependent shaping for a constant worst-case ankle velocity, such that energy injection is minimized across varying velocity trajectories. The second constraint, $P_{2,j}$, minimizes $\frac{\partial H - \partial \tilde{H}}{\partial \psi}$, which provides control torque with respect to the unactuated global coordinate ψ . The assumed worst-case magnitudes of non-passive inputs mimic software-enforced saturation limits.

C. GRF Shaping Constraint Implications on Performance

The GRF shaping constraint has implications for controller biomimicry and system stability. With $w_g = 0$, the optimal solution is almost exclusively force amplification, as GRF shaping alone provides a biomimetic approximation of ankle torque across tasks [28]. Moreover, the ankle performs significant positive work during ambulation [53], so a strictly passive controller is not desirable for providing biomimetic torque assistance. With increasing w_g , our framework trades off biomimicry for stability by reducing the GRF shaping contribution in favor of model-based terms.

The first row of Fig. 2 illustrates the evolution of control torque trajectories (u) from unconstrained to fully constrained (i.e., no GRF shaping), where the second row illustrates the corresponding GRF shaping control contributions ($\tilde{B}A^T \lambda$). The unconstrained controller is dominated by force amplification contributions during stance. As constraints on GRF shaping increase, its contribution decreases and responsibility for approximating control torque is traded to the basis functions of the system angles. The basis functions display worse performance in matching torque peak magnitudes across diverse tasks, as in previous work [41]. However, basis functions are still able to approximate the general shape of the ankle torque due to a few consistent relationships between ankle kinematics and kinetics. For instance, the foot angle ϕ provides a good indication of ground inclination when the foot is flat during midstance, and the global shank angle ψ provides a good indication of phase for ambulatory tasks [54], [55].

We perform leave-one-task-out and leave-one-subject-out cross-validation to verify that GRF suppression and biomimetic performance can generalize across tasks and subjects excluded from training. For leave-one-subject-out validation, we exclude all data (i.e., strides from all tasks) from a

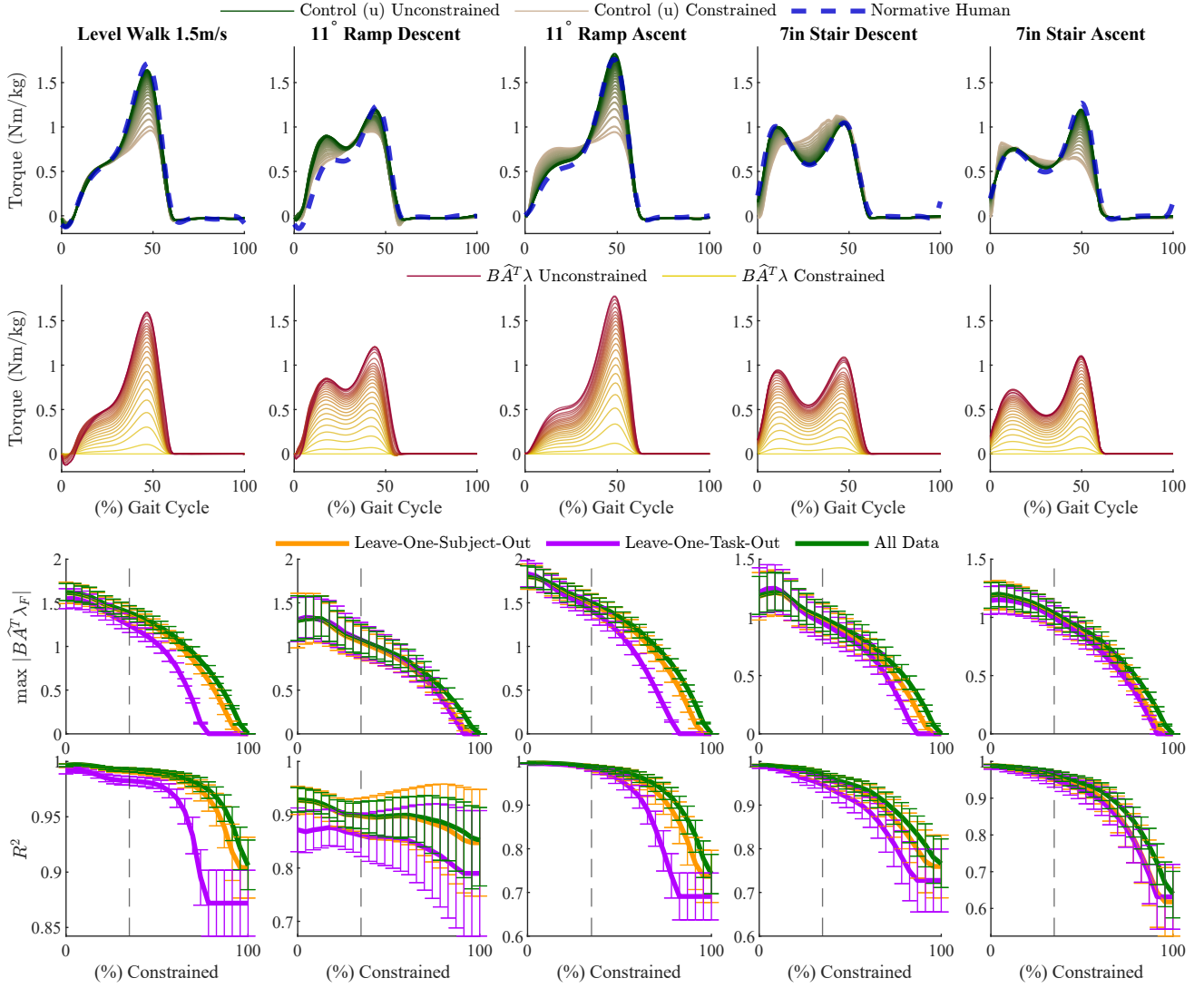


Fig. 2. Simulated effects of increasing GRF shaping constraints on controller biomimicry and stability for subjects and tasks in a normative dataset used for training [50]. Each column describes a unique task. The top row illustrates the evolution of control torque, u , from unconstrained to fully constrained (i.e., no GRF shaping), and the second row illustrates the corresponding GRF shaping contribution, $B\hat{A}^T\lambda$, for an inter-subject average stride across tasks. This simulation demonstrates how increasing constraints suppress the GRF shaping contribution while overall control torque maintains a biomimetic shape with diminishing torque peaks. The third row shows the maximum magnitude of the GRF shaping term for each task, where the solid line represents the mean and bars represent the confidence interval. The trends in $\max|B\hat{A}^T\lambda_F|$ illustrate that the constraint on non-passive GRF effort injection effectively decreases the size of the invariant set (a function of the worst-case GRF magnitude) and generalizes to tasks and subjects withheld from training. The bottom row displays the correlation between control torque and normative human torque, where the solid line represents the mean and bars represent the confidence interval. The correlation trends illustrate how biomimicry is maintained even as GRF contribution is reduced, particularly in the 0-50% constrained region, demonstrating the framework’s ability to reduce energy injection while maintaining performance by trading torque approximation from force amplification terms to basis functions of the system angles. The dashed grey line indicates the controller level of constraint used in the human subject experiment.

single subject during training and validate on the excluded subject’s average strides. For leave-one-task out validation, we exclude all data from each task type during training and validate on the excluded task. For instance, leave-one-task-out validation for 1.5m/s level walking uses coefficients trained without any examples of level walking (i.e., 0.5m/s, 1m/s, and 1.5m/s walking removed from training). We treat descent and ascent directions as separate tasks; for example, leave-one-task-out validation for ramp descent excludes both inclination examples of ramp descent while including ramp ascent. We present these cross-validations alongside results from subjects and tasks included in the training data (i.e., “All Data”), representing performance for in-distribution control inputs.

The third row of Fig. 2 shows the maximum magnitude of the GRF shaping term, $\max|B\hat{A}^T\lambda|$, for intra-subject average strides across select tasks. Across all tasks, $\max|B\hat{A}^T\lambda|$ decreases with increasing constraint for subjects and tasks left out of the training data, indicating that the optimization constraint is able to generalize to diverse ambulation activities. The magnitude of the GRF shaping term decreases faster during leave-one-subject-out and leave-one-task-out validation compared to training with all data. This accelerated decrease likely occurs because training with fewer data samples gives the scalar weight w_g greater influence on the cost function. However, since w_g is a tunable parameter, the specific magnitude of w_g needed to achieve a certain constraint is not critical.

The reduction in $\max |\hat{B}\hat{A}^T\lambda|$ improves the stability of the system, as the size of the invariant set depends on the worst-case magnitude of GRF shaping (33). While optimizing the controller with a limited set of stride examples across common tasks prevents us from making global behavioral guarantees, the leave-one-subject-out and leave-one-task-out cross validation demonstrates that the optimization still effectively reduces $\max |\hat{B}\hat{A}^T\lambda|$ for tasks and subjects excluded from training. Further, the confidence intervals on $\max |\hat{B}\hat{A}^T\lambda|$ narrow with increasing constraints for all training data and for excluded subjects and tasks. This consistent maximum GRF shaping reduction indicates that our framework achieves a robust representation of the input space and can effectively reduce non-passive effort contributions for out-of-distribution tasks and subjects without introducing high variability.

We also assess simulated controller biomimicry via the coefficient of determination (R^2) shown in the fourth row of Fig. 2. Following the standard definition, we define

$$R^2 = \left(\frac{1}{N-1} \sum_{i=1}^N \left(\frac{C_i - \mu_C}{\sigma_C} \right) \left(\frac{H_i - \mu_H}{\sigma_H} \right) \right)^2, \quad (16)$$

where C is the control torque, H is the normative human torque, μ is the mean, σ is the standard deviation, and N is the number of time-normalized data points per stride. We prioritize correlation as a metric of biomimicry as previous work demonstrated similar reductions in metabolic cost between static ankle torque profiles and personalized assistance profiles after several hours of training [56]. Correlation remains roughly consistent under 0-50% GRF constraint for in-distribution samples and left-out subjects, indicating that the framework successfully trades control torque contributions from GRF shaping to basis functions while maintaining biomimicry and generalizing across subjects. This trade-off highlights a benefit of the optimal framework leveraging basis functions, allowing tuning of the GRF contribution without significantly impacting controller performance. However, the system angles exhibit higher inter-subject and inter-task variability, as demonstrated by lower correlation for left-out tasks and larger confidence intervals under high GRF constraint. Thus, suppressing the GRF shaping entirely is not desired, as it provides critical information to achieve inter-subject and inter-task biomimicry in a self-contained ankle exoskeleton.

D. Power Leak Passivity

The energy injection through the power leak port can be constrained by specifying $w_p > 0$. At the ankle, however, force amplification terms dominate the controller because the ankle moment can be effectively approximated using the nGRF and COP. While this constraint may have limited utility at the ankle, the ability to modulate power leak energy injection could prove more valuable for proximal joints. If sufficiently large magnitudes of w_g and w_p are specified, torque contributions through both the GRF shaping and power leak ports would be completely suppressed. A completely passive controller would function solely as a non-linear spring, with behavior determined exclusively by the actuated angle via $\mathcal{B}_\theta \omega_\theta$.

IV. HARDWARE IMPLEMENTATION

This section details the ankle exoskeleton hardware and controller implementation. We empirically show the stability difference between controllers with unconstrained and partially constrained force amplification contributions to show the practical implications of the optimization constraints.

A. Ankle Exoskeleton Overview

The controller was implemented on the bilateral, backdrivable ankle exoskeleton system shown in Fig. 1 (modified from [57]). Briefly, torque is applied by an actuator with a brushless DC motor and integrated 9:1 planetary gearset (CubeMars AK80-9) with custom motor electronics (Dephy, MA), enabling 25Nm peak applied torque and low dynamic backdrive torque [58]. This inherently backdrivable quasi-direct drive actuator allows the user to remain in control of their movement, making it well-suited for energy shaping control based on the user's voluntary kinematic inputs. The actuator is located coaxial with the sagittal ankle joint and is secured to a reinforced boot and the user's shank via two aluminum sheet-metal uprights. The torque is transferred to the user via a carbon-fiber plate in the sole of the boot (Dephy, MA) and a 3D-printed shank cuff with aluminum sheet-metal reinforcement. Each unilateral exoskeleton is controlled by a Raspberry Pi 5 with 8 GB RAM housed in a 3D-printed electronics box on the proximal upright and is powered by a 24V, 2Ah lithium-ion battery (Kobalt) carried by the user on a waist-belt. Each unilateral exoskeleton (including the structural components, actuator, electronics, and sensor suite) weighs 1.25 kg and a single US men's size 10 reinforced boot adds 0.45 kg. Each battery (with attachment components) adds 0.64 kg at the waist.

B. Controller Implementation

The controller runs independently on each unilateral exoskeleton, relying on its self-contained sensor suite (Fig. 1). Inertial measurement units (IMUs) are mounted on the shank and foot to measure the global link angles and velocities (3DM-GX5-25, Lord MicroStrain). As the exoskeleton structure is not perfectly rigid, some flexion in the structure occurs as torque is applied, resulting in a discrepancy between the motor encoder angle and the human ankle angle. For the control law, we calculate the relative ankle angle θ as the difference between the IMU orientations instead of the motor encoder angle to provide a more accurate angle measurement with respect to the rigid body shank and foot model. The angular velocities are filtered with 2nd-order, low-pass Butterworth filters with 6Hz cut-off frequencies. The link angles are not filtered beyond the IMUs' onboard Kalman filter.

Force feedback is obtained via a pressure-sensing insole with 8 force-sensitive resistors (FSRs) placed under the user's foot in the boot (Actisense, IEE, Luxembourg). The FSRs approximate the nGRF, which is normalized to the interval $[0, 1]$ with end-points representing swing phase and single-support stance phase calibrated for each user, respectively. The COP is approximated as the weighted sum of the resistance

measured by each individual FSR, where each resistance is weighted by the sensor's anterior-posterior position relative to the ankle. The resistance measured by each individual FSR is filtered with a 2nd-order, low-pass Butterworth filter with 15Hz cut-off frequency.

The torque commanded to the actuator is the mass-normalized predicted torque (15) multiplied by the user's mass and user-specific assistance fraction. The assistance fraction was chosen such that the peak applied torque saturates at the peak actuator torque during the most strenuous tasks (e.g., fast walking and ramp ascent). We leverage a previously validated open-loop torque model accounting for compliance, friction, and inertia in the transmission to achieve accurate torque tracking [59]. To respect the actuator thermal limits, the torque is tapered when necessary based on an actuator thermal model [40]. The controller operates at 500Hz.

C. Empirical Stability Comparison

The integration of force amplification (FA) into the optimal energy shaping framework introduces the inherent risk of unstable oscillations from the positive feedback FA terms. The FSR does not directly measure a change in GRF induced by the commanded ankle torque because the FSR is inside the user's boot. However, in the case of low ankle impedance the commanded ankle torque can induce a change in the user's kinematics, influencing the location of the COP or the magnitude of the nGRF measured on the discrete FSR cells. For example, when a user is seated, the stiffness of the ankle relaxes, though the GRF sensors can still register up to 50% ground contact. A change in the measured nGRF or COP changes the commanded torque and consequently the user's kinematics, which can induce an oscillation around the ankle equilibrium angle, with the human ankle acting like a spring.

We qualitatively explored this oscillatory phenomenon in a demonstration with a single user. The user was seated on an armless, backless stool height-adjusted such that the user's knees were at a 90° angle when seated. The user was wearing a unilateral exoskeleton, with the instrumented leg placed at an angle under the stool such that the ball of the foot is in contact with the ground (Fig. 3). Thus, the COP was located at the ball of the foot, with the FSRs measuring the GRF due to the weight of the shank. To induce a change in the measured GRF, the user abruptly relaxed their plantarflexor muscles, which dropped their heel toward the floor. Fig. 3 shows the time-series control torque response to this disturbance with an unconstrained FA controller and a partially constrained FA controller. In both cases, the disturbance initially caused a "bang-bang" oscillation across the GRF threshold at 3% bodyweight, below which the GRF is set to zero to account for signal noise during swing. However, this initial oscillation described by hybrid dynamics eventually entered the continuous dynamics region for both controllers, though this transition was faster for the partially constrained FA controller. In the continuous region, the unconstrained FA controller maintained a steady-state oscillation, whereas the partially constrained FA controller stabilized to a constant equilibrium.

This qualitative comparison for a single able-bodied user demonstrates a practical implication of the FA constraints

in the optimization framework. In pilot testing, researchers empirically selected ~30% GRF restriction on the controller, indicated by the dashed vertical gray lines in Fig. 2. This partially constrained controller demonstrated an acceptable trade-off between biomimicry and the ability to suppress this oscillation scenario. Although this scenario obeys the continuous dynamics assumed in our stability proof, oscillatory transients (as seen in Fig. 3) can occur while converging to the invariant set. Moreover, oscillation can still occur within the invariant set. With unconstrained FA, the invariant set is large in the case of low joint impedance relative to the magnitude of the disturbance. The partially constrained controller restricts the size of the invariant set via a constraint on the maximum magnitude of the GRF disturbance, reducing the size of potential oscillations under similar GRF sensor measurements and human joint impedance conditions. However, the risk of oscillations in certain configurations cannot be completely eliminated when incorporating FA control contributions.

V. HUMAN SUBJECT EXPERIMENT METHODS

This section presents the methods for a study with $n=10$ human participants wearing bilateral ankle exoskeletons. We validate the controller biomimicry across ADLs and evaluate the effect of biomimetic assistance on human joint kinetics.

A. Experimental Protocol

Ten able-bodied individuals (33.3 ± 13.0 years, 68.3 ± 8.1 kg, 1.7 ± 0.1 m) consented to participate in the study approved by our Institutional Review Board. In a single testing session, participants first acclimated to the exoskeleton assistance and then completed the experimental trials in bare (no-exo) and exoskeleton-powered-on conditions (exo), where the order was alternated between participants. The exo condition consisted of the circuit followed by dedicated ADLs, and the no-exo condition consisted of only the dedicated ADLs.

1) *Acclimation*: Before donning the exoskeleton, participants performed three repetitions of stair ascent/descent and sit-stand movements in the reinforced boots to establish baseline speeds. The mean self-selected speeds were enforced across exo and no-exo conditions. After donning the exo, participants walked overground at incrementally increasing assistance fractions until the assistance saturated at 25Nm. Participants then acclimated to ramp ascent/descent, stair ascent/descent, sit-stand, stand-sit, and crouching with the exo.

2) *Circuit*: Participants completed five alternating clockwise (CW) and counter-clockwise (CCW) repetitions on a multi-terrain circuit to test the controller's instantaneous task-adaptive capabilities. The CW direction consisted of sit-to-stand, level ground, ramp incline, stair descent and stand-to-sit. The CCW direction consisted of sit-to-stand, stair ascent, ramp decline, level ground, and stand-to-sit. The circuit was set to the tallest configuration, with 7-inch step height and 14.1° ramp inclination, to achieve the largest variation in ADLs.

3) *Dedicated ADLs*: Participants walked on a level treadmill at three speeds (0.75m/s, 1m/s, and 1.25m/s) and on a 5° and 10° incline and decline at 1m/s for a minute each.

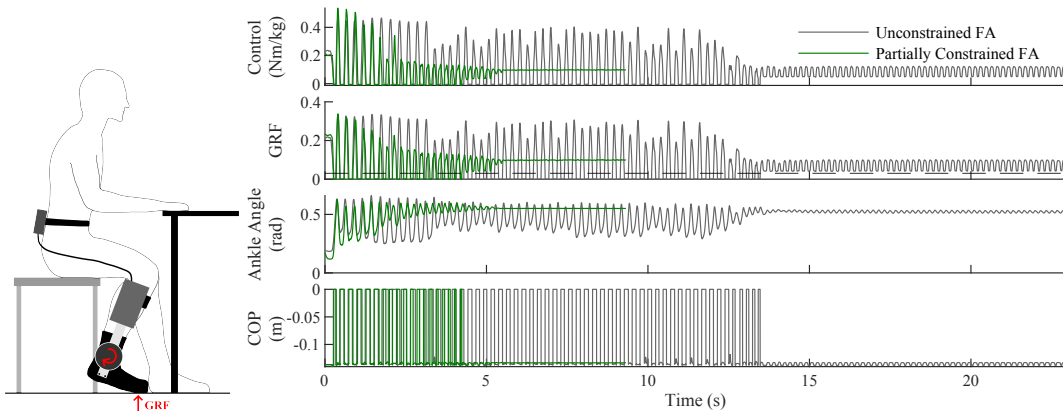


Fig. 3. (left) Seated configuration for empirical evaluation of ankle exoskeleton control instability; (right) Important exoskeleton signals for two (separate) controllers following similar perturbations, with an unstable oscillation for unconstrained force amplification (FA) control contributions and a stabilized response for partially constrained FA control contributions. The dashed black line indicates the GRF threshold at 3% bodyweight, below which the GRF is set to zero. The COP is set to zero with the same threshold as the GRF.

All treadmill tasks were completed on a split-belt force-instrumented treadmill (Bertec, OH). Participants ascended and descended a five-step force-instrumented staircase (Kistler, MI) with 7-inch step height five times in each direction to obtain 10 steady-state stride samples. Heelstrikes on the staircase were paced with a metronome according to the user’s selected stair ascent and descent paces to enforce consistent average velocity across each stride for exo and no-exo conditions and remove a source of variability when comparing human joint torque and power metrics. Participants completed stand-to-sit and sit-to-stand ten times each from a backless stool, with the stool height adjusted such that each participant’s knees were at a 90° when seated. Each participant’s feet were placed on forceplates (AMTI, MA) during the sit-stand cycles, with the stool located directly behind the forceplates. Finally, participants performed a crouching task ten times, simulating picking a light object off the floor.

B. Data Collection and Processing

The participants’ kinematics were collected using motion capture at 250Hz (Vicon, NY), and ground reaction forces from the forceplates were collected at 1kHz. Participants wore skin-tight long-sleeve and long-pant motion capture suits (OptiTrack, OR) with motion capture markers on the lower-limbs and torso. As the reinforced boots obscure the ankle’s bony landmarks and the exoskeleton structure can comply relative to the human ankle joint, the positions of the medial and lateral malleolus markers were calculated in post-processing via a rigid body transformation relative to markers on the shank. The ankle marker transformation was calculated using markers placed on the skin of the participant’s medial and lateral malleolus prior to donning the reinforced boots.

The inverse kinematics and inverse dynamics were processed using the generic “3DGaitModel2392” in OpenSim 4.5. Subject-specific models were scaled using the subject mass without the exoskeleton, and two separate models were customized for processing no-exo and exo trials. In the no-exo subject-specific models, the masses of the reinforced boot and FSR were added to the mass of the left and right talus. In the exo subject-specific models, the no-exo model was modified to

add the mass and inertia of the exoskeleton components—the masses of all exoskeleton structural components and sensors distal to the ankle joint center were added to the mass of each talus and the masses of all exoskeleton structural components proximal to the ankle joint center were added to the mass of each tibia. The actuator was modeled as a point mass with the inertial properties characterized in previous work [58] and rigidly attached to each ankle joint.

All strides were parsed heelstrike to heelstrike and time-normalized. Steady-state strides across dedicated ADLs were parsed using the forceplates, and circuit strides were parsed using the FSR. Treadmill tasks only include strides on the right tread forceplate due to a faulty load cell in the left forceplate. To calculate the human torque contribution during exoskeleton trials, we subtracted the torque commanded to the actuator from the net ankle torque calculated via inverse dynamics. This operation assumes perfect transfer of the commanded torque to the human through the rigid shank cuff and boot attachments, as there is no practical way to directly measure the torque imparted to the human.

We evaluate the controller featuring approximately 30% FA restriction, indicated by the dashed vertical gray lines in Figure 2. The stability properties of this controller were empirically demonstrated in Section IV-C. Controller biomimicry across dedicated ADLs was evaluated by comparing the mass-normalized control torque to the mass-normalized human torque calculated for the same stride. Controller biomimicry across task transitions was evaluated by comparing the mass-normalized control torque in the circuit test to mass-normalized inter-subject average human torque from a normative dataset [50], as the entire circuit was not instrumented with forceplates to calculate the user’s biological torque.

C. Statistical Analysis

We analyzed the association of peak human torque with condition (no-exo, exo) using a linear mixed model in R. The outcome of the linear mixed model was mass-normalized intra-subject mean peak biological ankle torque for each condition. The fixed effect was condition, and all models incorporated a random intercept for participant. The R formula for the linear

mixed model, fit to all data to determine the association of condition with peak torque across all tasks, was

$$\text{PeakTorque} \sim \text{Condition} + (1|\text{Participant}). \quad (17)$$

We fit separate linear mixed models for each task using the same R formula (17). We corrected for multiple comparisons (12 tasks) using the Bonferroni correction [60]. The interaction of biological sex and participant mass was considered in secondary analyses with the linear mixed model

$$\text{PeakTorque} \sim \text{Condition} + \text{Sex} \cdot \text{Mass} + (1|\text{Participant}). \quad (18)$$

We separately analyzed the secondary outcomes—mean log-transformed peak negative power, mean log-transformed peak positive power, and mean log-transformed absolute lower body work—using similar linear mixed models (17) with the same fixed and random effects to determine the percent change associated with condition. All kinetics were mass-normalized.

VI. HUMAN SUBJECT EXPERIMENT RESULTS

In this section, we present results of an experiment with ten able-bodied human subjects in no-exo and (bilateral) exo conditions. We evaluated controller biomimicry and human joint kinetic outcomes across various ADLs for a controller with partially restricted force amplification (FA) contributions.

A. Controller Biomimicry

With partially restricted FA control contributions, the task-agnostic controller approximated biomimetic torque across all tasks without user- or device-specific training data. Figure 4 shows the average mass-normalized control torques and corresponding human user torque across the steady-state activities (for which inverse dynamics could be calculated). Across all activities, the controller demonstrates a strong correlation with the instantaneous human torque ($R^2 = 0.83$) and low root-mean-squared error (RMSE) (0.17 Nm/kg). The mean correlation coefficients with confidence intervals (CIs) and the mean and standard deviation RMSE for each task are summarized in Table I.

TABLE I
STEADY-STATE CONTROLLER BIOMIMICRY

Task	Coeff. Determination (16) (R^2 [lower CI, upper CI])	RMSE (Nm/kg)
0.75m/s Level	0.91 [0.90, 0.91]	0.14 ± 0.05
1m/s Level	0.89 [0.89, 0.90]	0.15 ± 0.05
1.25m/s Level	0.87 [0.86, 0.87]	0.18 ± 0.06
5° Incline	0.86 [0.85, 0.86]	0.19 ± 0.05
5° Decline	0.95 [0.94, 0.95]	0.15 ± 0.08
10° Incline	0.86 [0.85, 0.87]	0.19 ± 0.06
10° Decline	0.93 [0.93, 0.94]	0.14 ± 0.07
Stair Ascent	0.82 [0.80, 0.86]	0.19 ± 0.07
Stair Descent	0.75 [0.73, 0.78]	0.26 ± 0.08
Sit to Stand	0.48 [0.44, 0.52]	0.14 ± 0.07
Stand to Sit	0.55 [0.51, 0.59]	0.17 ± 0.07
Crouch	0.45 [0.42, 0.49]	0.20 ± 0.11
All	0.83 [0.83, 0.84]	0.17 ± 0.07

Controller biomimicry varies across the different ambulatory and non-ambulatory tasks. Treadmill tasks demonstrate the highest correlation ($R^2 = 0.89$) with a narrow confidence

interval and mean RMSE of 0.16 Nm/kg. Stair tasks have a slightly lower correlation ($R^2 = 0.79$) with a wider confidence interval and mean RMSE of 0.23 Nm/kg. It is important to note that correlation measures are influenced by substantial differences in torque magnitudes across activities. The sit-stand, stand-sit, and crouch tasks have the lowest mean correlation ($R^2 = 0.49$), though these tasks have very small torque magnitudes compared to ambulation tasks. The mean RMSE of 0.17 Nm/kg for these non-ambulatory tasks falls within the RMSE range of ambulatory tasks, indicating that the controller biomimicry is comparable.

The task-agnostic controller also effectively adapts ankle torque across task transitions without explicit task knowledge or mode switching. Fig. 5 shows the average mass-normalized control torques across the circuit alongside average human torques from a normative dataset for corresponding tasks [50]. Despite differences in peak magnitudes, the control torque profiles accurately reflect the trends observed in human torque patterns, with increased early stance support peaks during stair tasks relative to walking tasks, and slightly smaller early stance peaks during ramp tasks.

B. Human Torque Reductions

With biomimetic torque assistance, participants reduced their peak plantarflexion torque during all steady-state tasks. Compared to no-exo, the exo assistance significantly reduced peak plantarflexion by 0.24 Nm/kg on average, corresponding to an average 19.1% reduction in ankle torque. Fig. 6 shows the average mass-normalized human torque in the no-exo and exo conditions together with the mass-normalized exo applied torque for all tasks. All peak torque reductions were significant (17) with the exception of sit-to-stand and crouch, as summarized in Table II, where ** and *** represent statistically significant differences with $p < 0.01$ and $p < 0.001$, respectively, as per our linear mixed model with $n=10$ and n.s. represents not significant.

TABLE II
CHANGE IN HUMAN ANKLE TORQUE FOR NO-EXO TO EXO

Task	Peak Change (Nm/kg)		
	Effect Size	[lower CI, upper CI]	
0.75m/s Level	-0.24	[-0.28, -0.20]	***
1m/s Level	-0.27	[-0.30, -0.24]	***
1.25m/s Level	-0.27	[-0.31, -0.24]	***
5° Incline	-0.29	[-0.32, -0.25]	***
10° Incline	-0.29	[-0.34, -0.23]	***
5° Decline	-0.27	[-0.30, -0.23]	***
10° Decline	-0.23	[-0.29, -0.18]	***
Stair Ascent	-0.22	[-0.30, -0.14]	**
Stair Descent	-0.28	[-0.34, -0.22]	***
Stand to Sit	-0.11	[-0.16, -0.07]	**
Sit to Stand	-0.02	[-0.06, 0.02]	n.s.
Crouch	-0.14	[-0.24, -0.04]	n.s.
All	-0.24	[-0.27, -0.22]	***

During level walking, the exo assistance decreased peak plantarflexion torque by 0.24 Nm/kg to 0.27 Nm/kg, corresponding to a 17.4% to 19.5% reduction in ankle torque. Incline treadmill tasks each achieved torque reductions of 0.29 Nm/kg, corresponding to 18.1% (10°) and 19.8% (5°) reductions in ankle torque. Decline tasks demonstrated 0.23 Nm/kg and 0.27 Nm/kg reductions in peak plantarflexion

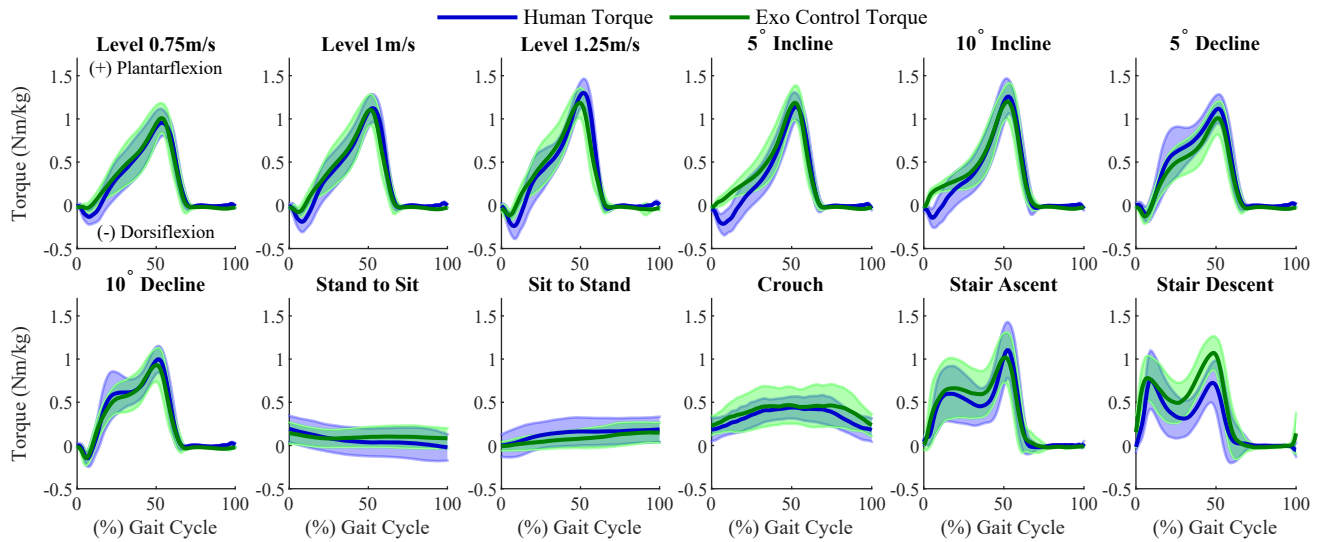


Fig. 4. Mass-normalized experimental control torque and simultaneous mass-normalized experimental human torque calculated via inverse dynamics for steady-state activities. The solid lines and shaded regions represent mean and standard deviation, respectively, of strides across all subjects.

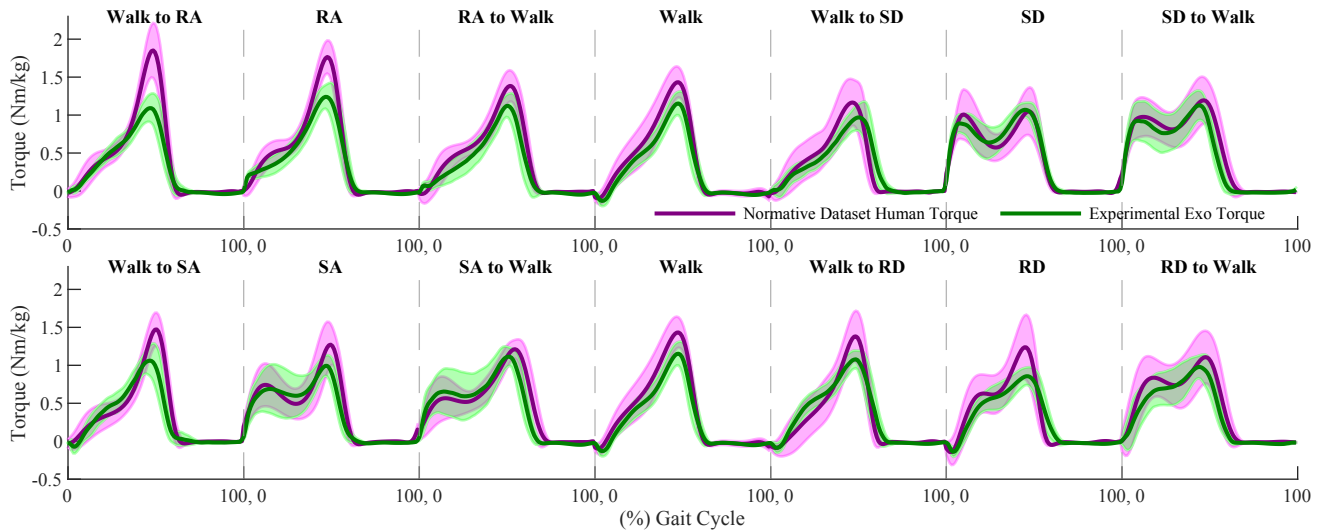


Fig. 5. Mass-normalized experimental control torque and average mass-normalized human torque from a normative dataset [50] across circuit task transitions. The circuit included continuously varying terrains for walking, ramp ascent (RA), ramp descent (RD), stair descent (SD), and stair ascent (SA). The solid lines and shaded regions represent inter-subject mean and standard deviation, respectively.

torque, corresponding to 18.5% and 18.8% reductions for 10° and 5°, respectively. Participants decreased their peak plantarflexion torque by 0.22 Nm/kg during stair ascent and by 0.28 Nm/kg during stair descent, corresponding to 14.7% and 22.3% reductions in ankle torque, respectively. Torque reductions during treadmill and stair tasks are consistent across subjects, as illustrated by the line plots of intra-subject mean peak torque in Fig. 7. Participants experienced the least reduction in joint torque during sit-stand, stand-sit, and crouch tasks, with reductions of 0.02 Nm/kg to 0.14 Nm/kg, though these tasks have the lowest normative torque magnitudes. These modest torque reduction magnitudes correspond to 1.5%, 35.5%, and 17.5% reductions in ankle torque for sit-stand, stand-sit, and crouch, respectively.

C. Human Energetics

As a secondary outcome, torque assistance led to reductions in peak negative and peak positive power at the ankle joint.

Compared to no-exo, the magnitude of peak negative power was reduced by 36.1% across all tasks. Peak positive power was reduced by 2.9% across all tasks, though this reduction is not statistically significant. Table III shows the peak positive and negative power reductions by task, where a negative value indicates a reduction in magnitude for exo compared to no-exo with significance defined in Sec. VI-B. Fig. 6 shows the mean and standard deviation ankle power for each task, with large reductions in peak negative power and small mean decreases in peak positive power for all ambulatory tasks with the exception of decline walking and sit-to-stand.

The torque assistance had mixed effects on total lower-body energetics. We consider absolute lower-body work (i.e., the integral of absolute power summed over the hip, knee, and ankle) as a measure of the holistic biomechanical effect of the exoskeleton inertia and assistance. Across all tasks, the exoskeleton yielded a non-significant 2.2% increase in absolute work. Table III shows the effect of condition on

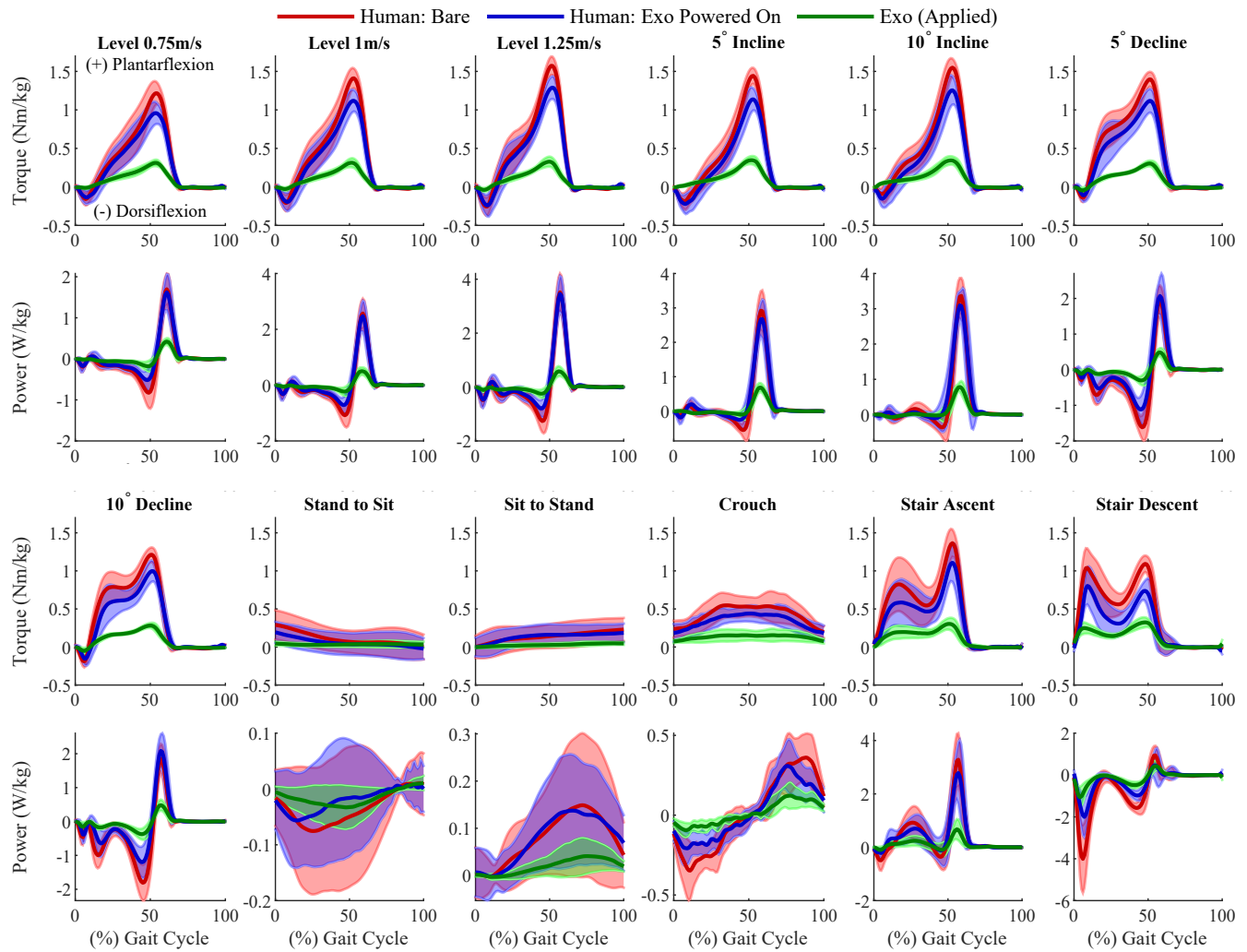


Fig. 6. Mass-normalized human ankle torque and power trajectories for no-exo (red) and exo powered on (blue) conditions, together with mass-normalized exo applied torque (green, corresponding to exo control torque in Figure 4).

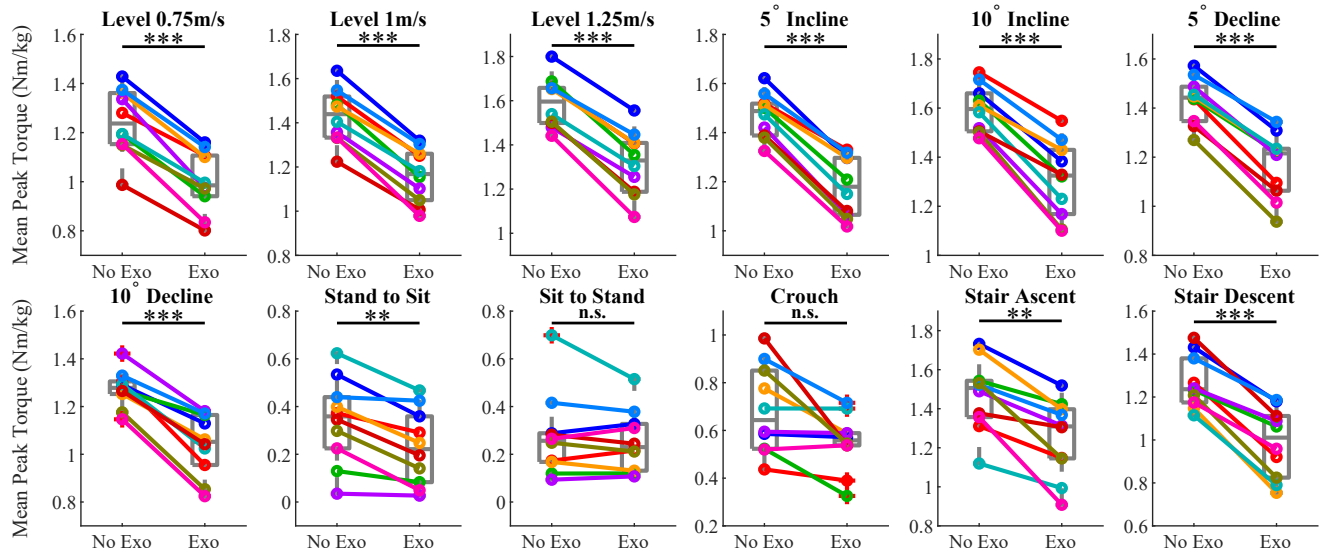


Fig. 7. Distribution of participant intra-subject mean mass-normalized peak human ankle torque for no-exo and exo powered on conditions. The exo consistently reduced the human peak ankle torque, where all reductions were statistically significant with the exception of sit-to-stand and crouch; ** and *** represent statistically significant differences with $p < 0.01$ and $p < 0.001$, respectively, as per out linear mixed model with $n=10$; n.s., not significant.

TABLE III
CHANGE IN HUMAN ENERGETICS FOR NO-EXO TO EXO

Task	Peak Positive Power (% Change)			Peak Negative Power (% Change)			Absolute Work (% Change)		
	Effect	[lower CI, upper CI]		Effect	[lower CI, upper CI]		Effect	[lower CI, upper CI]	
0.75m/s Level	-5.2	[-17.2, 6.8]	n.s.	-41.9	[-52.8, -30.9]	***	1.9	[-3.3, 7.2]	n.s.
1m/s Level	-2.7	[-13.8, 8.4]	n.s.	-33.6	[-41.7, -25.4]	***	3.1	[-0.8, 7.1]	n.s.
1.25m/s Level	-2.4	[-9.0, 4.3]	n.s.	-35.4	[-44.1, -26.8]	***	3.4	[0.7, 6.1]	n.s.
5° Incline	-6.5	[-16.1, 3.0]	n.s.	-42.4	[-58.0, -26.8]	**	3.1	[0.4, 5.8]	n.s.
10° Incline	-7.2	[-15.8, 1.5]	n.s.	-38.5	[-60.7, -16.2]	n.s.	2.0	[-0.4, 4.4]	n.s.
5° Decline	9.6	[-1.6, 20.8]	n.s.	-30.0	[-38.7, -21.3]	***	0.7	[-4.0, 5.3]	n.s.
10° Decline	12.0	[0.2, 23.8]	n.s.	-37.7	[-45.2, -30.2]	***	4.1	[0.8, 7.5]	n.s.
Stair Ascent	-13.0	[-28.3, 2.4]	n.s.	-60.0	[-90.4, -29.5]	*	-0.9	[-9.8, 8.0]	n.s.
Stair Descent	-38.4	[-64.8, -11.9]	n.s.	-69.1	[-87.8, -50.3]	***	-4.8	[-10.3, 0.6]	n.s.
Stand to Sit	-7.1	[-53.5, 39.3]	n.s.	-26.2	[-56.7, 4.4]	n.s.	2.0	[-1.5, 5.4]	n.s.
Sit to Stand	7.2	[-18.9, 33.3]	n.s.	1.5	[-60.0, 62.9]	n.s.	4.0	[0.9, 7.2]	n.s.
Crouch	-10.9	[-31.1, 9.4]	n.s.	-31.8	[-48.4, -15.2]	*	-2.4	[-6.6, 1.8]	n.s.
All	-2.9	[-12.6, 6.7]	n.s.	-36.1	[-43.8, -28.4]	***	2.2	[-0.1, 4.7]	n.s.

absolute lower-body work for each task. The maximum total absolute work penalty with the exoskeleton is a 4.1% increase during 10° decline. The hip and knee joints are responsible for this increase in absolute lower-body work, with the ankle demonstrating a significant -10.8% change in total work (CI [-16.8%, -4.7%], $p < 0.01$) across all tasks for exo compared to no-exo. Stair tasks and crouch demonstrated total lower-body work reductions with exo assistance, with stair descent achieving an average 4.8% reduction.

VII. DISCUSSION

In this paper, we developed an optimization-based framework combining complementary stability and biomimicry strengths of model-based energy shaping and force amplification, respectively, and implemented an ankle exoskeleton controller with partially restricted force amplification (FA). We discuss the advantages of this framework, the controller biomimicry and joint kinetic outcomes from a study with ten able-bodied participants, study limitations, and future work.

A. Controller Optimization Framework

The controller optimization framework fits a task-agnostic model, defined with basis functions of the system angles, to examples of normative ankle torque across ambulatory tasks. The basis functions are selected as trigonometric functions of the shank, ankle, and foot angles, as these angles capture intuitive relationships with ankle torque across strides [41]. For example, the shank angle approximates a phase variable for treadmill tasks [54], [55] and the foot angle captures information about the ground inclination. However, instantaneous kinematic feedback available to a self-contained ankle exoskeleton is insufficient to discriminate between treadmill tasks and stair tasks with our hand-engineered basis functions [41], motivating the incorporation of the FA terms for improved controller biomimicry.

The explicit formulation of this control law has advantages over hidden (“black-box”) learning-based approaches, such as neural networks. Although neural networks might find better nonlinear relationships between kinematics and ankle torque with long input sequences [21], [61], our framework enables direct and transparent modification of the control law to achieve specific, desired behaviors. For instance, we engineered the swing-phase basis functions to restrict overfitting to

smooth training data. Moreover, by selecting basis functions from the lowest-frequency terms in the Fourier series, we inherently restrict high-frequency content in the control torque and bound the output, permitting direct application of the control signal without requiring additional filtering. Each control contribution (including both passive and non-passive model elements) remains interpretable, facilitating clear analysis of the sources of torque behaviors. Theoretical analysis of the non-passive control components identifies which elements may introduce destabilizing effects, thereby informing optimization constraints and ultimately supporting the stability and safety of the human-exoskeleton system. Notably, our optimization problem is convex, which can be solved quickly for the global optimum (without hyper-parameter tuning) to enable fast iteration (such as future customizations to patient populations). While cost function weights may be tuned (as with any learning-based approach), the explicit derivation of all basis functions enables intuitive and rapid adjustment of cost function weights to achieve targeted behaviors (such as robust dorsiflexion during swing) with guaranteed optimality.

While we have shown strong biomimetic performance with our optimization-based controller, the ankle moment can be reasonably calculated simply as a scalar-weighted product of the nGRF and COP [28]. This straightforward approach could arguably negate the need for a complex framework with nonlinear basis functions. Our optimization accommodates this approach through a bias term in the set of trigonometric basis functions that shape GRF components, allowing for a solution of purely direct FA (i.e., nGRF-COP). However, ankle moment estimation error using only the nGRF increases for activities with high tangential GRF (tGRF) components for braking and acceleration (e.g., ramp incline/decline). By modeling FA as a nonlinear function of system angles, we achieve somewhat more accurate moment estimation as tGRF increases. Notably, the optimal solution sends the coefficient scaling the linear bias FA term to zero—even under no GRF shaping constraints—indicating that a nonlinear model improves biomimicry. This finding aligns with previous work fusing direct FA of EMG-sensed muscle activation with a nonlinear joint angle model to improve performance [25].

The primary advantage of this optimization-based framework over naive direct force amplification is the ability to smoothly constrain the force amplification control contribution

in favor of passive, model-based terms while maintaining biomimicry. Our constraints on the magnitude of GRF shaping are particularly important for ankle exoskeleton control due to the positive feedback loop between ankle torque and GRF. We empirically demonstrated an unstable oscillation for an unconstrained controller in a seated configuration under conditions of low biological ankle impedance. Under similar initial conditions, a controller with partially constrained GRF contributions is stabilized, exhibiting only a transient oscillatory response before settling. This improved stability is attributed to a reduced maximum magnitude of the GRF-induced disturbance, which restricts the size of the invariant set in which oscillations can occur. As a result, the closed-loop system with partially constrained FA maintains asymptotic stability to a smaller invariant set, effectively suppressing sustained oscillations in this configuration. Although (small) oscillations are still possible in certain configurations, the size of the invariant set can be tuned based on the FA constraint to find an acceptable tradeoff with biomimicry. This framework allows FA contributions to safely and effectively improve the differentiation of assistance between tasks in a self-contained ankle exoskeleton [41].

B. Controller Biomimicry

With approximately 30% GRF restriction, the controller provided biomimetic torques across tasks both inside and outside the training data. Notably, the controller did not require device- or user-specific training data as commonly used in neural network approaches [5], [21]; our method was inherently robust to kinematic changes relative to normative training data due to inter-subject variation or exo assistance. We prioritized biomimicry in level walking by weighting stride samples more heavily in the cost function during training. Treadmill tasks demonstrated the highest correlation, with a narrow confidence interval and low RMSE standard deviation, indicating consistent torque biomimicry across subjects. The controller successfully approximates increases in push-off torque peaks with increasing walking speed, though perfect alignment is constrained by limits on the GRF feedback. The controller also effectively approximated dorsiflexion peaks at heelstrike for all treadmill tasks except incline, though this discrepancy involved small torque magnitudes. Higher variation in stair tasks is likely attributed to higher inter-subject variation in stair climbing gait strategies, which can vary from calf-dominant to quad-dominant, whereas treadmill tasks are generally calf-dominant during the push-off phase [62]. Importantly, the controller applied torque of the correct sign and magnitude for stand-to-sit, sit-to-stand, and crouch tasks, demonstrating that the controller can generalize to unseen, non-steady tasks despite being trained exclusively on ambulation tasks.

As the control law is trained on examples of steady-state activities, we qualitatively evaluated the controller biomimicry on transition strides between circuit tasks. Task transitions occur primarily during swing, but ankle kinetics do not deviate much in swing due to the low inertia of the foot. The ankle begins to exhibit transition behavior in mid-stance [63], corresponding to subtle differences in stance torque during task

transition strides (Figure 5). While the controller generally exhibits reduced peak torque magnitudes compared to the normative dataset, direct comparison is challenging as participants differ between studies and gait strategies (e.g., walking speed and stride length) were not controlled. It is important to note that the torque peak magnitudes for steady-state tasks (Figure 4), where corresponding instantaneous human inverse dynamics could be calculated, demonstrate better torque peak matching. This qualitative comparison demonstrates that our explicit, task-agnostic controller continuously adapts transitions appropriately without explicit task knowledge.

C. Human Torque Reductions

Biomimetic ankle torque assistance yielded consistent peak plantarflexion torque reductions across all tasks and participants with little acclimation time. This torque reduction corresponds to a decrease in compressive forces on the joint articular surface [1], where compressive forces are linked to joint cartilage damage and osteoarthritis [43]. Reducing joint torque could potentially decrease load-related pain for individuals with osteoarthritis [7]–[9], [45] or prevent fatigue that increases a joint’s vulnerability to overuse and osteoarthritis initiation or progression [4], [44]. While previous work has demonstrated ankle torque reductions with biomimetic assistance during constant-speed treadmill walking [64], [65], to our knowledge this is the first demonstration of torque reductions across multiple diverse tasks using a single controller.

The subject- and task-specific torque reductions appear to be related to the assistance torque magnitudes. We investigated the association of peak human torque with condition and the interaction of participant biological sex and mass (18). Participant mass had a significant ($p < 0.05$) effect on the mass-normalized peak torque reduction across all tasks, indicating that the assistance provided a larger benefit (i.e., torque reduction) for individuals with lower body mass. Biological sex and the interaction of sex and mass were not significant effects. This finding is reasonable, as the assistance fraction was tuned for each participant to saturate at the maximum actuator assistance of 25Nm. An actuator with a higher peak torque could be used to obtain larger torque reductions for individuals with higher mass, though the inertial penalty of a heavier device should be considered. The modest torque reduction for sit-stand, stand-sit, and crouch tasks is likely due to minimal normative human joint torque across these tasks. Further, it is unclear what degree of biomimicry is necessary to achieve beneficial joint torque reductions. Among ambulation tasks, stair descent shows the lowest control torque correlation, likely due to a higher torque peak during push-off compared to human torque. Despite this discrepancy, stair descent achieved the highest average joint torque reduction, suggesting that lower correlation is not problematic and that larger torque peak magnitudes are beneficial for increased torque reduction [56].

D. Human Energetics

Biomimetic torque assistance reduced users’ peak positive and peak negative ankle power across most tasks. Across treadmill tasks, peak positive power reduction was attributed to the

consistently decreased plantarflexion torque. The modest peak positive power reductions, compared to larger peak negative power reductions, were likely due to the consistently increased peak ankle velocities at push-off for the exo condition compared to no-exo. Increased push-off velocity also led to higher peak positive power in the exo condition for ramp decline, which notably corresponded to decreased peak positive hip power at swing initiation. Conversely, during swing initiation for level treadmill tasks, ankle peak positive power decreased slightly without a noticeable change in peak positive hip power. These variations in power generation across joints might have resulted from individual gait strategies to leverage the assistance, where the added exo push-off power (with no significant difference in human push-off power) assisted with propelling the added mass of the device into swing. Alternatively, the experiment might not have allowed sufficient acclimation time for participants to optimally leverage the powered assistance while minimizing proximal joint effort from the inertial penalty of the device. However, our study was not intended to evaluate optimal gait efficiency, as optimal benefit can take considerable training [56].

The significant decrease in negative power was attributed to decreases in ankle torque and small decreases in peak dorsiflexion angle across tasks. It is possible that reducing negative power at the joint may not be optimal for reducing metabolic cost [65], though previous work demonstrated that biomimetic assistance can achieve metabolic cost reductions [28], [64]. In this study, we prioritized reductions in joint-level effort measures at the assisted joint and did not assess metabolic cost between conditions. These negative power reductions contribute to reduced joint loading, as eccentric contractions produce larger muscle forces than concentric contractions for equivalent angular velocities [66].

We present the change in absolute lower-body work as a holistic measure of the change in both eccentric and concentric muscle contraction. We generally found a small ($< 4\%$), non-significant increase in absolute lower-body work with the exoskeleton. This holistic increase was likely due to the distal inertial penalty in the form of increased positive work, primarily during swing phase. As the entire experiment was completed in a single testing session, participants experienced short acclimation time. Although participants consistently demonstrated torque reduction benefits after little acclimation, future work could investigate the effects of extended acclimation time on joint power and work measures.

E. Other Limitations and Future Work

This study had several limitations. Due to fixed force-plate locations, we were limited to quantitatively evaluating controller biomimicry and human joint kinetic outcomes for steady-state ambulatory tasks and isolated non-steady movements (i.e., sit-stand). We could only qualitatively evaluate biomimicry across activity transitions relative to a normative dataset, where a direct comparison is difficult due to a lack of consistent controls between experiments. Further, we only quantitatively evaluated a subset of tasks. Future work could quantitatively evaluate controller biomimicry for a wider range

of tasks and task transitions. Due to time constraints, we also did not evaluate a passive condition, as we prioritized showing benefit with the active exoskeleton over the no-exo condition. A passive condition could evaluate the increase in lower-body work due to the inertial penalties of the device to isolate the energetic benefit of the powered assistance.

While our unconstrained control design was dominated by FA terms during stance (with the nonlinear basis functions accounting for the unmeasured tGRF), direct FA is less straightforward for proximal joints where the tGRF component is more significant and swing torques have higher magnitudes with more variability across tasks. Including both model-based and force-amplification components allows our framework to extend to more proximal joints, either in cooperation with the ankle or independently [40]. Future work could apply this framework to modular exoskeleton configurations (as in [57], [58]) and explore the practical implications of the power leak optimization constraint, which was not investigated here.

Future work could evaluate the benefits of biomimetic ankle torque assistance across tasks for broader populations. Impaired populations may be more motivated to leverage the biomimetic assistance to reduce their ankle effort than able-bodied participants (due to pain or motor control limitations). A pilot study with participants experiencing pain due to chronic ankle osteoarthritis demonstrated promising results, with larger peak torque and peak positive power reductions alongside acute pain reductions after similar acclimation time [9]. As biomimetic assistance can reduce painful joint loading while improving mobility metrics (e.g., walking speed and joint symmetry) [6], [9], future work should continue to explore the benefits for users with osteoarthritic pain. Individuals with healthy ankle joints could also benefit from a powered ankle exoskeleton by leveraging the assistance to reduce effort at an impaired proximal joint, like the hip [62].

VIII. CONCLUSION

This work combined the advantages of energy-shaping control and force amplification through a data-driven optimization approach. By combining the stability benefits of energy shaping with the superior biomimicry of force amplification, our controller overcomes key challenges of existing approaches by maintaining explainability and providing guarantees for user safety. Experimental results from a cohort of ten able-bodied human participants demonstrate that our framework achieves strong biomimetic performance across diverse ambulatory and non-ambulatory activities, both inside and outside of the training data. The high correlation between the controller torque and human torque ($R^2 = 0.83$ across all tasks) validates our approach's ability to approximate biological ankle torque patterns. The biomimetic assistance enabled significant reductions in peak plantarflexion torque across all steady-state tasks, averaging 0.24 Nm/kg (19.1%) overall. These consistent torque reductions can decrease compressive loading of the joint articular surface, suggesting that this device could decrease osteoarthritic pain and overuse injuries. Overall, this optimization-based extension of energy-shaping control offers a promising direction for developing partial assistance ex-

oskeletons that provide safe, intuitive, and effective assistance across the diverse activities of daily living.

APPENDIX

In this section, we show the closed-loop system converges to a forward-invariant set for a simple impedance-controlled human input. For notational clarity, we define functions $\max_\lambda(A)$ and $\min_\lambda(A)$ to find the maximum or minimum eigenvalue λ , respectively, of a matrix $A \in \mathbb{R}^{n \times n}$. We now prove Theorem 1.

Proof 2: The assumptions (12) and (13) of the Theorem are matrix conditions; we first verify their feasibility by exhibiting scalar sufficient conditions. Any α satisfying

$$\alpha < \sqrt{\frac{\min_\lambda(K_p)}{\max_\lambda(M)}}, \quad \text{and} \quad (19)$$

$$\alpha < \frac{4 \min_\lambda(K_d)}{4 \max_\lambda(M) + (\max_\lambda(K_d) + |\bar{\kappa}|)^2 / \min_\lambda(K_p)} \quad (20)$$

also satisfies (12) and (13), where $\bar{\kappa} = \kappa(\bar{q})$ is the velocity-dependent shaping scalar at the human setpoint, per Sec. III. The first implication holds because $\alpha^2 \max_\lambda(M) < \min_\lambda(K_p)$ implies $\alpha^2 M \prec K_p$. The second holds because $(K_d - \bar{J})^T K_p^{-1} (K_d - \bar{J}) \preceq \max_\lambda(K_p^{-1}) (\max_\lambda(K_d) + |\bar{\kappa}|)^2 I$, as the nonzero singular values of $\bar{J}$ equal $|\bar{\kappa}|$. Both bounds admit a small positive α .

To analyze system stability, we define the state-space model with error state $x = [e^T, \dot{e}^T]^T$ and system dynamics

$$\dot{x} = Ax + \Lambda, \quad \text{where} \quad (21)$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K_p & -M^{-1}(K_d - \bar{J}) \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 0 \\ \delta \end{bmatrix},$$

and $\delta = M^{-1}(\hat{A}^T \lambda_F + B_x u_x)$ expresses the non-passive port inputs (GRF shaping and power leak) as an external disturbance. Here, (21) linearizes the target system (2) under the human input (10) about $\bar{q}$, where the interconnection term contributes the skew-symmetric $\bar{J}$ to the damping block, and higher-order terms are neglected. The workless constraint forces $A^T \lambda_F$ are omitted. During stance, both the trajectory and the setpoint satisfy the contact constraint, $a(q) = 0$ and $a(\bar{q}) = 0$, so expanding about $\bar{q}$ gives $A(\bar{q})e = 0$ and $A(\bar{q})\dot{e} = 0$ to first order. The error thus evolves in the null space of $A(\bar{q})$, orthogonal to the constraint forces; the swing phase has no contact constraints.

Let $\beta \geq \delta^T \delta$, where $\beta \in \mathbb{R}^+$ is a bound on the quadratic disturbance. We assume that software limits saturate the GRFs such that the shaping term $\hat{A}^T \lambda_F$ is bounded. The power leak term $B_x u_x$ is bounded by the bounded basis functions (Sec. III) under biomechanically bounded velocity magnitudes.

Consider the following candidate Lyapunov function,

$$\mathcal{W}(x) = \frac{1}{2} x^T W x, \quad \text{with } W = \begin{bmatrix} K_p & \alpha M \\ \alpha M & M \end{bmatrix}. \quad (22)$$

We establish positive-definiteness from the Schur complement:

$$W \succ 0 \iff \begin{cases} M \succ 0, \\ K_p - \alpha M M^{-1} \alpha M \succ 0, \end{cases} \quad (23)$$

with the first case true by positive-definiteness of the mass/inertia matrix and the second equivalent to the assumption (12). The function $\mathcal{W}$ has time-derivative

$$\dot{\mathcal{W}} = \frac{1}{2} x^T (WA + A^T W) x + x^T W \Lambda = \frac{1}{2} x^T \bar{W} x + x^T W \Lambda, \quad (24)$$

where

$$\bar{W} = \begin{bmatrix} -2\alpha K_p & -\alpha(K_d - \bar{J}) \\ -\alpha(K_d + \bar{J}) & 2\alpha M - 2K_d \end{bmatrix}. \quad (25)$$

Here, $\bar{J}$ canceled in the lower-right block, as $\bar{J} + \bar{J}^T = 0$.

Given the linearizing assumptions with constant matrices, we will show $\bar{W} \prec 0$, which will confine non-negative $\dot{\mathcal{W}}$ to a bounded region of the state space. By the Schur complement, the negative definiteness of $\bar{W}$ is equivalent to

$$\begin{cases} -2\alpha K_p \prec 0, \\ 2\alpha M - 2K_d + \alpha(K_d + \bar{J})(2\alpha K_p)^{-1} \alpha(K_d - \bar{J}) \prec 0, \end{cases}$$

with the first true by assumption that $K_p \succ 0$ and $\alpha > 0$, and the second equivalent to assumption (13), using the identity $(K_d + \bar{J})K_p^{-1}(K_d - \bar{J}) = (K_d - \bar{J})^T K_p^{-1} (K_d - \bar{J}) \succeq 0$, which is symmetric. With $\bar{W} \prec 0$, we now characterize the region of the state space in which $\dot{\mathcal{W}} \leq 0$ is guaranteed.

A sufficient condition for $\dot{\mathcal{W}} \leq 0$ is

$$\frac{1}{2} x^T \bar{W} x \leq -\max_\delta x^T W \Lambda, \quad (26)$$

where the right side represents the worst-case increase in $\dot{\mathcal{W}}$ resulting from a maximum value of the disturbance, δ . Because the scalar $x^T W \Lambda = \delta^T [\alpha M; M]^T x$ is linear in δ , its maximum over $\delta^T \delta \leq \beta$ is attained with equality by Cauchy-Schwarz,

$$\max_\delta x^T W \Lambda = \sqrt{x^T \bar{Q} x}, \quad \text{with } \bar{Q} = \begin{bmatrix} \alpha M \\ M \end{bmatrix} \beta \begin{bmatrix} \alpha M \\ M \end{bmatrix}^T \succeq 0, \quad (27)$$

so (26) is equivalent to

$$\frac{1}{2} x^T \bar{W} x \leq -\sqrt{x^T \bar{Q} x}. \quad (28)$$

Note that $x^T \bar{Q} x = \beta \|\alpha M e + M \dot{e}\|^2$ vanishes on the subspace $\dot{e} = -\alpha e$, where the disturbance $x^T W \Lambda = \delta^T [\alpha M; M]^T x$ also vanishes, so (28) holds there by $\bar{W} \prec 0$ alone.

Away from this subspace we bound each side by its extreme eigenvalue. The negative-definite matrix $\bar{W}$ gives

$$\frac{1}{2} x^T \bar{W} x \leq \frac{1}{2} \max_\lambda(\bar{W}) \|x\|^2, \quad (29)$$

and the disturbance magnitude is bounded by

$$\sqrt{x^T \bar{Q} x} \leq \sqrt{\max_\lambda(\bar{Q})} \|x\|. \quad (30)$$

Substituting (29) and (30) into (28), we obtain

$$\frac{1}{2} \max_\lambda(\bar{W}) \|x\|^2 \leq -\sqrt{\max_\lambda(\bar{Q})} \|x\|. \quad (31)$$

The nonzero eigenvalues of $\bar{Q}$ equal those of $\beta(1 + \alpha^2)M^2$, so $\max_\lambda(\bar{Q}) = \beta(1 + \alpha^2)(\max_\lambda(M))^2$. Simplifying in terms of $\|x\|$ then yields $\|x\| \geq r$ with

$$r := \frac{2\sqrt{\beta(1 + \alpha^2)} \max_\lambda(M)}{\min_\lambda(-\bar{W})}, \quad (32)$$

determined by the maximum bound on the disturbance (β) and the minimum human impedance (captured in $\bar{W}$).

Thus, for any disturbance satisfying $\delta^T \delta \leq \beta$, $\dot{\mathcal{W}} < 0$ whenever $\|x\| > r$, with $\dot{\mathcal{W}} \leq 0$ at $\|x\| = r$. As the level sets of $\mathcal{W}$ are ellipsoids, the ball $\{\|x\| \leq r\}$ itself need not be invariant. We therefore take the invariant set to be the smallest sublevel set of $\mathcal{W}$ containing this ball,

$$\mathcal{R} = \left\{ x : \mathcal{W}(x) \leq \frac{1}{2} \max_{\lambda} (W) r^2 \right\}. \quad (33)$$

Since $\mathcal{W}(x) \leq \frac{1}{2} \max_{\lambda} (W) \|x\|^2$, any x outside $\mathcal{R}$ satisfies $\|x\| > r$, with $\|x\| \geq r$ on its boundary, so by the sign of $\dot{\mathcal{W}}$ no trajectory exits $\mathcal{R}$: it is forward invariant [67]. By (29) and (30), $\dot{\mathcal{W}}$ is bounded above outside $\mathcal{R}$, uniformly in the disturbance, by a continuous function of $\|x\|$ that is strictly negative for $\|x\| > r$, so trajectories converge to $\mathcal{R}$ by a standard ultimate boundedness argument. Finally, $\frac{1}{2} \min_{\lambda} (W) \|x\|^2 \leq \mathcal{W}(x)$ on $\mathcal{R}$ yields the ultimate bound $\|x\| \leq r \sqrt{\max_{\lambda} (W) / \min_{\lambda} (W)}$, determined by β and the minimum human impedance. $\square$

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Katharine Walters (Student Member, IEEE) received the B.S. degree in mechanical engineering in 2022 from Michigan State University, East Lansing, MI, USA and the M.S. degree in robotics in 2025 from the University of Michigan, Ann Arbor, MI, USA, where she is currently working toward the doctoral degree in robotics with the Locomotor Control Systems Laboratory, Robotics Department.

Her research interests include control of robotic systems, human locomotion and biomechanics, and clinical outcomes of wearable assistive devices.



Gray C. Thomas (Member, IEEE) received the B.S. degree in engineering: robotics with the Olin College of Engineering, Needham, MA, USA, in 2012, and the Ph.D. degree in mechanical engineering from the University of Texas at Austin, Austin, TX, USA, in 2019.

He is an Assistant Professor with the Department of Mechanical Engineering, Texas A&M University, College Station, TX. He studies direct human control of physically interactive robots through a combination of strength amplification control, human modeling and estimation, and high-bandwidth design. His other research interests include force-feedback, series-elastic actuation, and robust control.



Robert D. Gregg (Senior Member, IEEE) received the B.S. degree in electrical engineering and computer sciences from the University of California at Berkeley, Berkeley, CA, USA, in 2006 and the M.S. and Ph.D. degrees in electrical and computer engineering from the University of Illinois at Urbana-Champaign, Champaign, IL, USA, in 2007 and 2010, respectively.

He was an Assistant Professor with The University of Texas at Dallas, Richardson, TX, USA. In 2019, he joined the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA, as an Associate Professor. He joined the Robotics Department, University of Michigan in 2022, where he is now a Professor. His research interests include the control of bipedal locomotion with applications to autonomous and wearable robots.